Lucia’s Analysis: Held & Soden without “hypothetical partials”

By David Evans, 16 Oct 2015

Lucia’s analysis is here.

Set-up

- One driver, CO_2, which Lucia, following H&S, represents by “$log_2 CO_2$” but here is represented here by “L” (as per my posts).
- One feedback, H_2O, which Lucia represents implicitly in the OLR and ASR functions R and S because $H_2O(T)$ depends only on T, while R and S are both functions of T.
- The ASR is

$$S = S(T)\,. \tag{1}$$

  Notice that although S is not directly dependent on T, it is depends on the feedbacks which in turn depend on T.
- Lucia writes the OLR as

$$R = R(T,L) = R_{\mathrm{p}}(T) + R(T,L) \tag{2}$$

  where $R_{\mathrm{p}}(T)$ is the OLR where feedbacks are held constant at the values of the current Earth and is independent of the drivers—i.e. $R_{\mathrm{p}}(T)$ is the OLR as temperature varies if all feedbacks and drivers are held constant. R is simply the OLR less $R_{\mathrm{p}}(T)$. Lucia sometimes writes “R_{pe}” for “R_{p}”.

Incremental Equation

Between steady states $\Delta S = \Delta R$, so, by Eq.s (1) and (2),

$$\frac{dS}{dT}\Delta T = \frac{dR_{\mathrm{p}}}{dT}\Delta T + \frac{\partial R}{\partial T}\Delta T + \frac{\partial R}{\partial L}\Delta L\ \ . \tag{3}$$

This is Lucia’s Eq. (5), her counterpart of H&S’s Eq. (6) or Eq. (2) in post 3. If there are no feedbacks then it becomes

$$0 = \frac{dR_{\mathrm{p}}}{dT}\Delta T + \frac{\partial R}{\partial L}\Delta L\,. \tag{4}$$

Analysis

By Eq. (4), the no-feedbacks ECS is

$$\Delta_0 = \frac{\Delta T}{\Delta L} = -\frac{\dfrac{\partial R}{\partial L}}{\dfrac{dR_{\mathrm{p}}}{dT}} \tag{5}$$

(beware, Lucia has the sign wrong on this, so I changed it here to what I presume she meant; Δ_0 as defined here is a positive number, hers as posted is negative). Lucia defines

$$\beta_{\mathrm{T}} = \frac{\dfrac{dS}{dT} - \dfrac{\partial R}{\partial T}}{\dfrac{dR_{\mathrm{p}}}{dT}} \tag{6}$$

so

$$\frac{1}{1-\beta_{\mathrm{T}}} = \frac{\dfrac{dR_{\mathrm{p}}}{dT}}{\dfrac{dR_{\mathrm{p}}}{dT} - \dfrac{dS}{dT} + \dfrac{\partial R}{\partial T}}. \tag{7}$$

By Eq. (3), the (with feedbacks) ECS is

$$\mathrm{ECS} = \frac{\Delta T}{\Delta L} = \frac{\dfrac{\partial R}{\partial L}}{\dfrac{dS}{dT} - \dfrac{dR_{\mathrm{p}}}{dT} - \dfrac{\partial R}{\partial T}} = -\frac{\dfrac{\partial R}{\partial L}}{(1-\beta_{\mathrm{T}})\dfrac{dR_{\mathrm{p}}}{dT}} = \frac{\Delta_0}{1-\beta_{\mathrm{T}}}. \tag{8}$$

Interpretations

By its definition R_{p} holds all feedbacks and drivers constant, so

$$\begin{aligned}\frac{dR_{\mathrm{p}}}{dT} = \left.\frac{\partial R}{\partial T}\right|_{\substack{\text{all feeedbacks}\\ \text{all drivers}}} &= \text{Planck feedback} = 3.2 \pm 0.1\ \mathrm{W\,m^{-2}}\ \text{per °C} \\ &= \frac{1}{\lambda_0} = \frac{1}{\text{Planck sensitivity}}.\end{aligned} \tag{9}$$

$\partial R/\partial L$ is the increase in OLR holding T and thus all feedbacks constants as one driver increases (and when generalized to where there is more than one driver, all the other drivers are held constant too), so

$$\frac{\partial R}{\partial L} = \left.\frac{\partial R}{\partial L}\right|_{\substack{\text{temperature}\\ \text{all feedbacks}\\ \text{all other drivers}}} = -D_{\mathrm{R,2X}} = -3.7\,[3.5, 4.1]\ \mathrm{W\,m^{-2}}. \tag{10}$$

Thus

$$\Delta_0 = -\frac{\dfrac{\partial R}{\partial L}}{\dfrac{dR_{\mathrm{p}}}{dT}} = \lambda_0 D_{\mathrm{R,2X}} = 1.16\,[1.08, 1.29]\ \text{°C}, \tag{11}$$

which is the no-feedbacks ECs just as per Eq. (8) of post 2.

Finally, by Eq.s (6) and (9)

$$\beta_{\mathrm{T}} = \frac{\dfrac{dS}{dT} - \dfrac{\partial R}{\partial T}}{\dfrac{dR_{\mathrm{p}}}{dT}} = \lambda_0 \left(\frac{dS}{dT} - \frac{\partial R}{\partial T} \right). \tag{12}$$

Varying T in S varies all the feedbacks but not the drivers, and S is not directly dependent on T – so only the single feedback variable can vary. Generalizing to the situation where there are multiple feedbacks, each feedback is varied in turn. Similarly with R, where holding L (and by implication, all drivers) constant in R while allowing T (and by implication all feedbacks) to vary means that all the drivers are held constant in the OLR, and since R_{p} captures the variation of OLR with temperature alone, only the feedback is being varied. Again, generalizing to multiple feedbacks means adding the variations due to each feedback in turn. Hence, if there are m feedbacks and G is S less R, then

$$\begin{aligned} \frac{dS}{dT} - \frac{\partial R}{\partial T} &= \left.\frac{\partial S}{\partial T}\right|_{\substack{\text{temperature} \\ \text{all drivers}}} - \left.\frac{\partial R}{\partial T}\right|_{\substack{\text{temperature} \\ \text{all drivers}}} \\ &= \sum_{i=1}^{m} \frac{\partial S}{\partial U_i}\frac{dU_i}{dT_{\mathrm{S}}} - \sum_{i=1}^{m} \frac{\partial R}{\partial U_i}\frac{dU_i}{dT_{\mathrm{S}}} \\ &= \sum_{i=1}^{m} \frac{\partial G}{\partial U_i}\frac{dU_i}{dT_{\mathrm{S}}} \\ &= f\,, \end{aligned} \tag{13}$$

where f is the total feedback parameter defined in Eq. (8) of post 3 and the U_i are the feedbacks. Thus

$$\beta_{\mathrm{T}} = f\lambda_0 \tag{14}$$

and, by Eq.s (8) and (11),

$$\mathrm{ECS} = \frac{\Delta_0}{1-\beta_{\mathrm{T}}} = \frac{\lambda_0}{1-f\lambda_0} D_{\mathrm{R,2X}} \simeq 2.5[1.24, 3.7]\ {}^{\circ}\mathrm{C} \tag{15}$$

in agreement with Eq. (18) of post 3.

Comments

Lucia's analysis works, which is hardly surprising considering she copied it out of H&S 2000 with notational variations. By omitting the feedback arguments from the ASR and OLR functions she has successfully transferred them implicitly to her new functions, S for the ASR, and R for the part of OLR that depends on feedbacks and drivers.

She merely moved the holding of all feedbacks and drivers constant to R_{p}, which she simply *defines* to be that part of the OLR that changes with temperature when all feedbacks and drivers are held constant. Well isn't that the partial derivative we've been talking about in post 2 and post 4, the Planck feedback? So we did all that running around just so Lucia could write it without a partial derivative symbol, but *define* it as holding feedbacks and drivers constant instead?

Her approach is considerably more complicated than the usual one, so why bother?

Lucia's False Claims

Each of her claims stems from her misconception that her formulation does not contain the traditional Planck feedback. On the contrary, her quantity R_p by definition holds all feedbacks and drivers constant and her formulation includes dR_p/dT, which is the traditional Planck feedback:

$$\frac{dR_p}{dT} = \left.\frac{\partial R}{\partial T}\right|_{\substack{\text{all feeedbacks}\\ \text{all drivers}}} = \text{Planck feedback} = \lambda_0^{-1} = 3.2 \pm 0.1\ \text{W m}^{-2}\ \text{per °C}. \qquad (16)$$

While "dR_p/dT" may not look like partial derivative because she rewrote the problem so that R_p was independent of feedbacks and drivers (she merely *defined* R_p as the OLR less that part of the OLR that depends on drivers and feedbacks), it is the partial derivative of the OLR with respect to surface temperature under the Planck conditions, i.e. while holding everything constant except tropospheric temperatures and OLR. It is still the Planck feedback, still estimated the same way, still holding all the feedbacks and drivers constant, still the same number. Semantic word game only.

Lucia makes four claims that are just restatements of her misconception:

1. Lucia claims that in her formulation, "partial differential are not taken holding "everything about the climate" constant." Nonsense. Her quantity dR_p/dT is a partial derivative of the OLR in which feedbacks (and thus virtually every climate variable) and drivers are held constant as temperature changes—see Eq. (16). It is the same partial derivative, the Planck feedback, that I talked about in my posts 2, 3, and 4 as not existing and being empirically unverifiable.
2. Lucia claims that her "math doesn't claim to hold all these constant [everything else apart from OLR and surface temperature — including humidity, clouds, gases, lapse rates, the tropopause, and absorbed sunlight] while taking partial differences". Nonsense. Her quantity dR_p/dT does precisely that; see previous claim.
3. Lucia claims that "the partials in [her formulation] contain no dependent variables, and so are not "hypothetical"." Nonsense. Her formulation contains the Planck feedback dR_p/dT, which relies on the existence of climate states where everything is held constant except temperature and OLR, which is plainly impossible. See post 4.
4. Lucia claims that "In my formulation, my Plank sensitivity does not contain any partial derivatives holding [everything except tropospheric temperatures (which all change uniformly) and OLR] constant". Presumably Lucia means *the* Planck sensitivity, namely

$$\text{Planck sensitivity} = \lambda_0 = \left(\frac{dR_p}{dT}\right)^{-1} = \left(\left.\frac{\partial R}{\partial T}\right|_{\substack{\text{all feeedbacks}\\ \text{all drivers}}}\right)^{-1} = 0.31 \pm 0.01\ \text{°C W}^{-1}\,\text{m}^2. \qquad (17)$$

More nonsense. Her Planck sensitivity holds precisely those variables constant.

Lucia goes on to make claim about my post, but because they all stem from her misconception above they are all false.

The final and obvious thing to say about Lucia's claims: If they were true, the climate scientists would have been using her formulation and "her Planck sensitivity" long ago, because it would obviously be far more realistic than having to hold everything else constant. But they don't.

The Comments on Lucia's Post

A big "thank you" to the commenters on Lucia's post who dared disagree with her — Angech ("There is something not right about an artifact developing on the implicit but not the explicit explanation that suggests the mapping is not analogous."), jim2, and others.

Nick Stokes seems to have swallowed Lucia's kool aid on this post, failing to notice that her claims are just legerdemain. Getting into the spirit of things that Lucia is creating, he says "There is no "IPCC version" involved here. Evans has made up a model that he calls the "basic" "conventional" model, but he gives no links to justify that usage." Hardly, Nick:

- From post 1: "Over the next two posts, we will present the basic model properly, in full detail, in keeping with the leading theorists and the dominant textbook."
- Then in post 2, "The conventional basic climate model is partially described by two foremost theorists, Isaac Held and Brian Soden, in their paper of 2000 [1], and more completely on pages 163–165 of the "gold standard" of climate textbooks, Raymond Pierrehumbert's *Principles of Planetary Climate* [2] (recommended if you want to know establishment climate science). We get the parameter values from the IPCC's latest assessment report from 2013, AR5 [3]." where [1^] Held, I. M., & Soden, B. J. (2000). Water Vapor Feedback and Global Warming. *Annu. Rev. Energy. Environ.*, 25:441–75. [2^] Pierrehumbert, R. T. (2010). *Principles of Planetary Climate.* Cambridge: Cambridge University Press.

Nick quotes AR5 on other more complex models but fails to note that the blog series is about *the* conventional *basic* climate model. There is only one, the really basic one, in the climate textbooks, and so on. Nick, read post 1 for why this is relevant and important.