The Notch-Delay Solar Theory: Excerpts released with the spreadsheet

Dr David Evans, 14 July 2014

6 Form of the Solar Model

The *form* is the relationship between the input and output, specifying all the internal pieces of the model and their connections. The form defines the model. The form is expressed here as an equation, which contains parameters whose values are unknown (but estimated in the next section).

The development of the form of the solar model is guided by the empirical transfer function, considers rates of heat accumulation, and notes the time scales of atmospheric processes.

6.1 The Notch

The most prominent feature in the empirical transfer function is the "notch", or "negative peak", at frequencies around 11 years. The amplitude of the transfer function is about 1.0°C per W/m^2 at the shoulders of the notch (at about 6 and 16 years), but only about 0.2°C per W/m^2 in the trough of the notch.

The notch is consistent with a cursory inspection of the TSI and temperature changes since 1610:

- Figure 1 and Figure 2 show that the TSI undergoes recurring prominent changes on a quasi-cyclic basis, with a "period" of about 9–14 years but typically around 11 years. These quasi-cyclic changes dominate the record of TSI changes, which therefore contains relatively large sinusoids with periods around 11 years.
- The corresponding temperature in Figure 1 and Figure 2 shows no such pattern; there isn't any dominant peaking every 9–14 years.

To put some numbers on it: TSI typical varies from trough to peak of a sunspot cycle by about 0.8 W/m^2. At the surface of the Earth, this is about 0.14 W/m^2 of unreflected TSI. If this was a long term change, the Stefan-Boltzmann equation would imply a change in radiating temperature of about 0.05°C, which would result in a change in surface temperature of likely about 0.1°C by the RATS multiplier (sections 6.4 and 7.2). The peaks only last for a year or two, so the low pass filter in the climate system (section 6.3) would reduce the temperature peak to somewhat below 0.1°C. The error margin of the temperature records is generally about 0.1°C, but Fourier analysis will usually find repetitive bumps down to a small fraction of the error margin, maybe a tenth. However these bumps are not quite regularly spaced, so the threshold of detectability would be a bit higher. In any case, we'd expect the temperature peaks to be detectable using the data and methods we have employed, though not by a huge margin.

(Later we propose a physical interpretation of the notch that implies a countering of the TSI warming, but of course such a countering would be very unlikely to completely cancel out the temperature effects of the TSI peak. But given that the margin for detection for the TSI peak alone is not great, it is credible that the mainly-countered TSI peaks are indeed not detectable.)

The notch was also later found, independently, by [Eschenbach, Solar Periodicity, 2014] using different means.

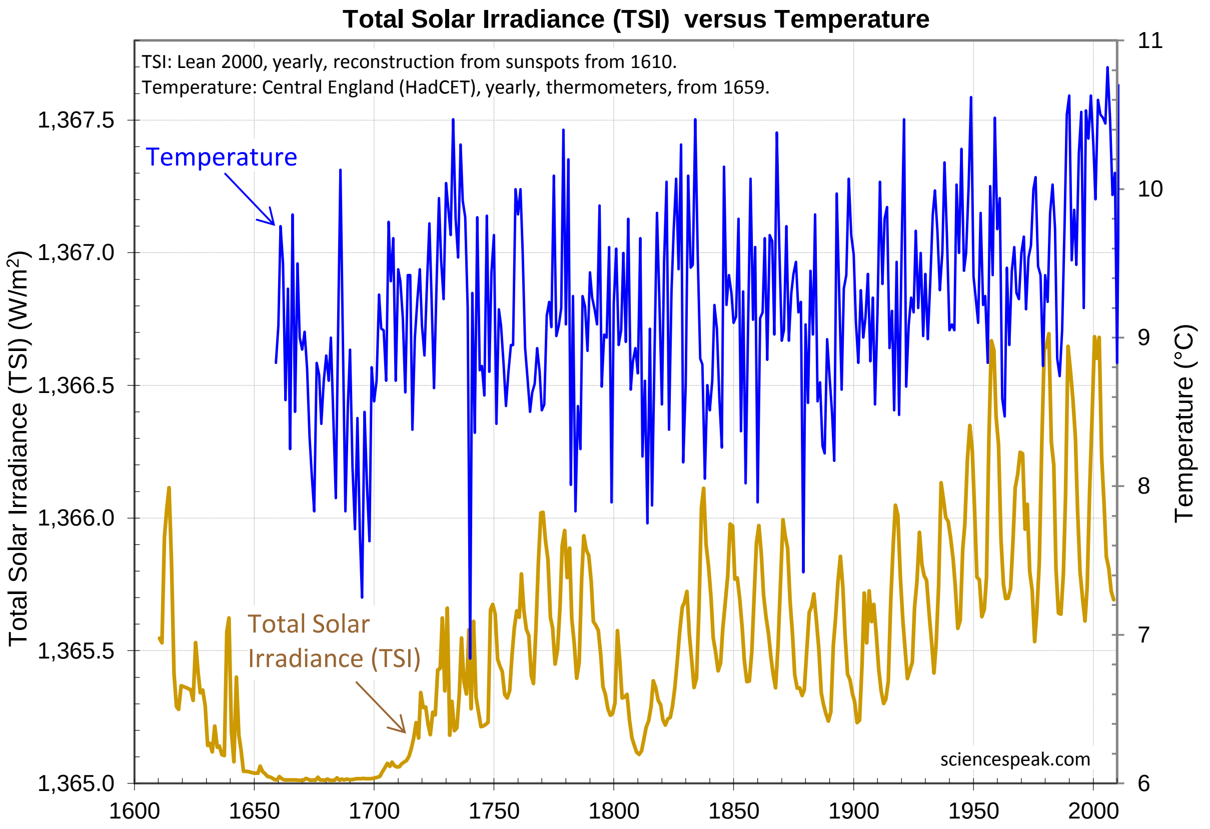

Figure 1: TSI and temperature compared, since recording of solar cycles began in 1610 AD. The prominent 11 year heartbeat of the TSI is not apparent in the temperature.

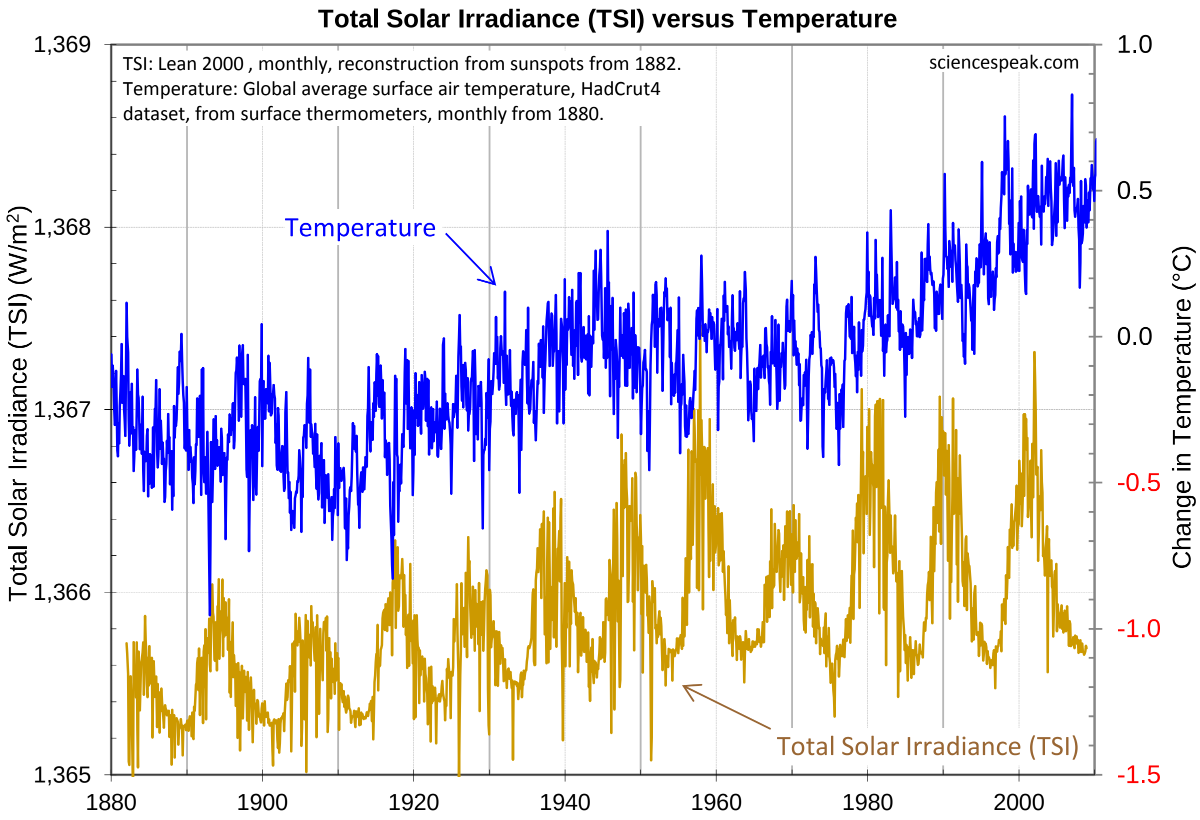

Figure 2: TSI and measured temperature monthly since Lean's TSI reconstructions became monthly in 1882. Again, the prominent 11 year sunspot "cycle" in the TSI is apparently absent in the temperature.

As discussed above, the solar subsystem is assumed to be linear and invariant and therefore it can be understood as transferring TSI sinusoids to temperature sinusoids, where what happens at each frequency is isolated from what happens at other frequencies. So either:

1. TSI changes have little effect on temperature changes. TSI sinusoids at all frequencies are associated with only small temperature sinusoids, that is, they are severely attenuated on their way to becoming temperature sinusoids. The temperature change record is almost entirely caused by non-solar factors and happens to have a flat spectrum.
2. TSI changes have a significant effect on temperature changes. TSI sinusoids around 11 years are being attenuated severely, but the TSI sinusoids at other frequencies are associated with temperature sinusoids of significant amplitudes.

The first option is in line with the CO_2 assumption, the CO_2 theory, and the resulting climate models. However on timescales of centuries or longer, something other than CO_2, presumably something solar and thus presumably associated with TSI change, is apparently a major determinant of temperature change. This implies that TSI sinusoids (or something associated with them) with frequencies of centuries or longer are not attenuated severely by the solar subsystem. So there must be some frequency above which they are severely attenuated, and below which they are not attenuated so much.

The second option is in line with the solar assumption and a significant role for TSI changes in causing or signaling temperature changes. We are entertaining that possibility in this paper, so we will assume for the purpose of model building that this is the case. This suggests the solar subsystem contains a low-quality notch filter that severely attenuates a moderately broad range of frequencies around 11 years but which passes other frequencies with little loss. (Notch filters in electronics are usually higher "quality", meaning they attenuate only a much narrower band of frequencies.)

The transfer function of the simplest type of notch filter that is also a linear invariant system is

$$H_{\mathrm{Notch}}(f) = \frac{f_Z^2 - f^2 + i\,2\,f\,f_Z \cos\theta_Z}{f_P^2 - f^2 + i\,2\,f\,f_P \cos\theta_P}, \quad f \geq 0, \tag{1}$$

where

- f_Z is the zero frequency, $f_Z > 0$. The zero period is $1/f_Z$.
- θ_Z is the zero angle, $\theta_Z \in [0, 90°]$ necessary for stability.
- f_P is the pole frequency, $f_P > 0$. The pole period is $1/f_P$.
- θ_P is the pole angle, $\theta_P \in [0, 90°]$ recommended for stability.

The notch frequency is close to f_Z, that is, $|H_{\mathrm{Notch}}(f)|$ is minimized when f is about equal to f_Z. The other parameters control the depth and width of the notch, and the relative heights of the shoulders (the amplitudes just above and below the notch frequency).

To illustrate this transfer function, we use parameter values that give a notch like the one in the empirical transfer function. To produce a notch at 11 years, the zero frequency f_Z is one eleventh of a cycle per year. The other three parameters give a similarly wide band of attenuated frequencies, a higher high-frequency shoulder, and a deeper sharper notch than in the empirical transfer function (whose notch, found by dividing two smoothed lines, is smoothed). In section 7 below we find sets of possible parameter values for the solar model, and one of our favorite sets is called "P25". We form the "P0" set of parameter values by rounding off the numbers in P25, and we use P0 to illustrate the model development throughout this section.

The notch filter of equation (1) is shown in the frequency domain in Figure 3. However it is very instructive to also view it in the time domain. To this end, the step response of the transfer function is calculated by applying the transfer function in equation (1) to the unit step function.

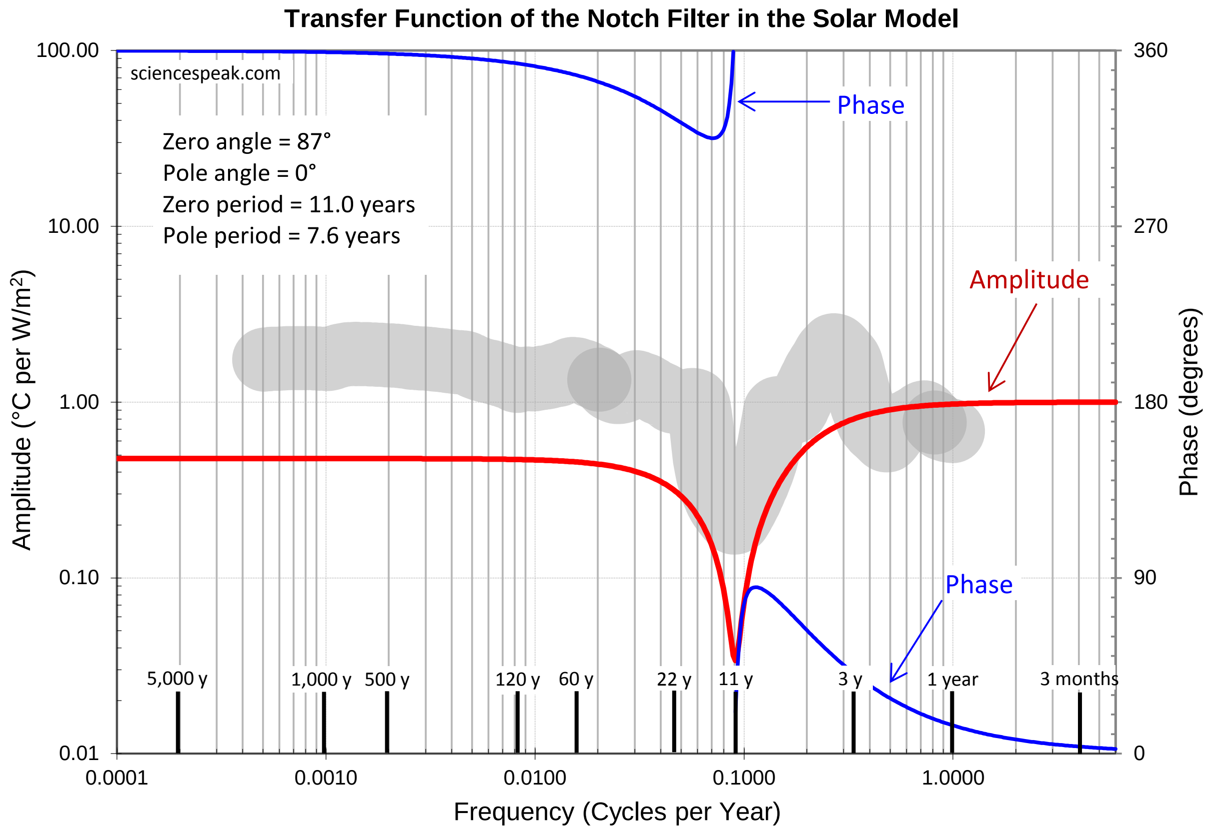

Figure 3: The transfer function of the notch filter in the solar model, showing equation (1) with the relevant parameter values from the P0 set of parameter values (determined later in section 7).

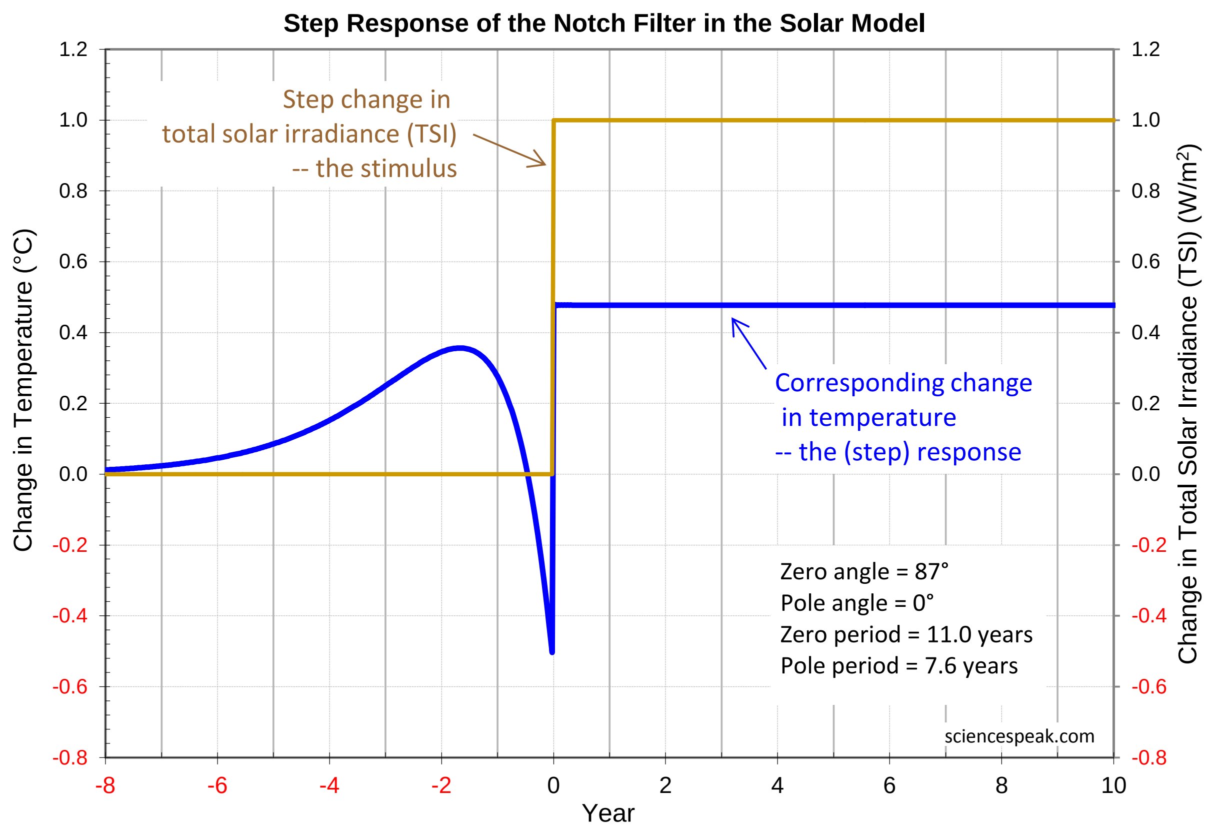

Figure 4: The step response of the notch filter in the solar model, corresponding to the transfer function in Figure 3. It is non-causal, that is, the response starts before the stimulus!

The step response of the notch filter is shown in Figure 4. The unit step in TSI occurs at the beginning of year 0, yet the step response, the corresponding change in temperature, begins at least five years *prior* to that, plainly violating causality! (A causal system is one where the effect comes *after* the cause—so its step response is zero for all times before the step in the input occurs.) This appears to be impossible, yet plainly there is a notch, so what gives?

6.2 The Delay

The non-causality of the step response of the notch filter shown in Figure 4 is not a fluke: in every set of notch parameters tried in equation (1) the step response is blatantly non-causal (see the *Climate.xlsm* spreadsheet). Notch filters are intrinsically non-causal.

When engineers design a filter whose response is the shape they want but which is non-causal because the response starts before the stimulus, and is therefore impossible to build, they simply include a delay with the filter. This moves the step response to the right in diagrams such as Figure 4. The combined filter-delay combination, being causal, is possible to build.

There does not seem to be any other possibility here. Given that there is clear evidence of notching, and that a notch filter necessarily has a non-causal step response, the only rational conclusion is that there must also be a delay. The notch *must* be accompanied by a delay, because the notch on its own is physically impossible.

With the parameters shown in Figure 4, the delay must be at least five years because the response is significant for at least five years before the stimulus starts. Changes in TSI are dominated by a quasi cycle whose average time between peaks is around 11 years, so an obvious candidate for the delay is 11 years. [Soon, 2009] found a ten year delay from TSI to sea surface temperature changes in the tropical Atlantic (see his Figure 4). [Moffa-Sanchez, Born, Hall, Thornalley, & Barker, 2014] find a lag of around 12 years from TSI to North Atlantic surface temperatures over the last thousand years (page 6 of the Supplementary Notes). [Usoskin, Schuessler, Solanki, & Mursula, 2004] found that the correlation coefficient between northern hemisphere temperature and reconstructed sunspot numbers from 850 AD was greatest when the temperature lagged the sunspot numbers by around 12 years—where the sunspot numbers were reconstructed from Be10 isotopes (see their Fig. 3). [Solheim, Stordahl, & Humlum, 2012] found that a lag of 11 years gives maximum correlation between solar cycle length and temperature. [Yoshimura, 1996] found that the TSI leads some index of the solar magnetic field by 10.3 years, and posited that the Sun could affect the Earth's climate "through two channels".

So let us add a delay filter into our solar model, in cascade with the notch filter. The delay filter simply delays the output of the solar model by a delay d. Its transfer function is

$$H_{\text{Delay}}(f) = \exp\left(-i\,2\pi fd\right), \quad f \geq 0\,. \tag{2}$$

The amplitude of the delay filter is unity at all frequencies, but its phase changes more and more quickly as frequency increases.

The delay is intrinsically due to or associated with the notch. If whatever causes the notch were separate from whatever causes the delay, the notchy part would be a non-causal system in its own right, which would be impossible. The notch filter and the delay thus exhibit themselves as parts of the same mechanism, and cannot be separated.

Some support for a delay can be found simply by comparing the temperature and the 11-year delayed TSI, as in Figure 5, and noting that there is some alignment of turning points.

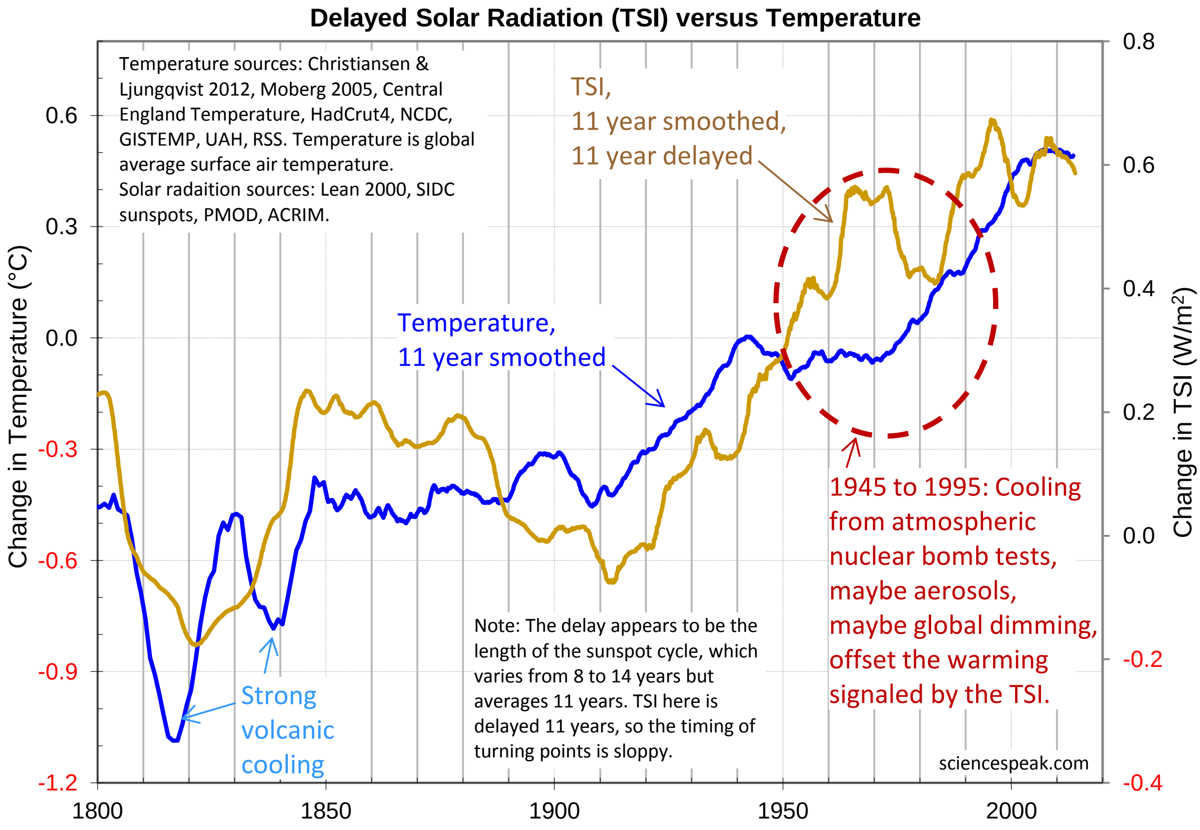

Figure 5: Comparing the composite TSI and composite temperature (Appendix K.2), where the TSI has simply been delayed by 11 years. Some turning points more or less align, suggesting that a delay around 11 year might exist. Later we find that various cooling influences could explain the failure of the temperature to follow the TSI up from 1945 to 1995, and in section 9.8 we find the delay may vary with the length of the solar cycle. The temperature data from 1850 to 1979 is predominately from land thermometers, and before 1850 is mainly proxy data.

6.3 The Low Pass Filter

Consider the system whose input is the record of changes in *unreflected* TSI, and whose output is the record of changes in the radiating temperature. By the basic physics of heat accumulation from the Sun and radiation to space, this system is a low pass filter and its transfer function is

$$H_{\mathrm{LPF}}(f) = \frac{m_L}{1 + i\left(f / f_{\mathrm{B}}\right)}, \quad f \geq 0, \tag{3}$$

where f_{B} is the "break frequency" and m_{L} is the "LPF multiplier". See Figure 6. This result does not rely on either the CO_2 or solar assumptions.

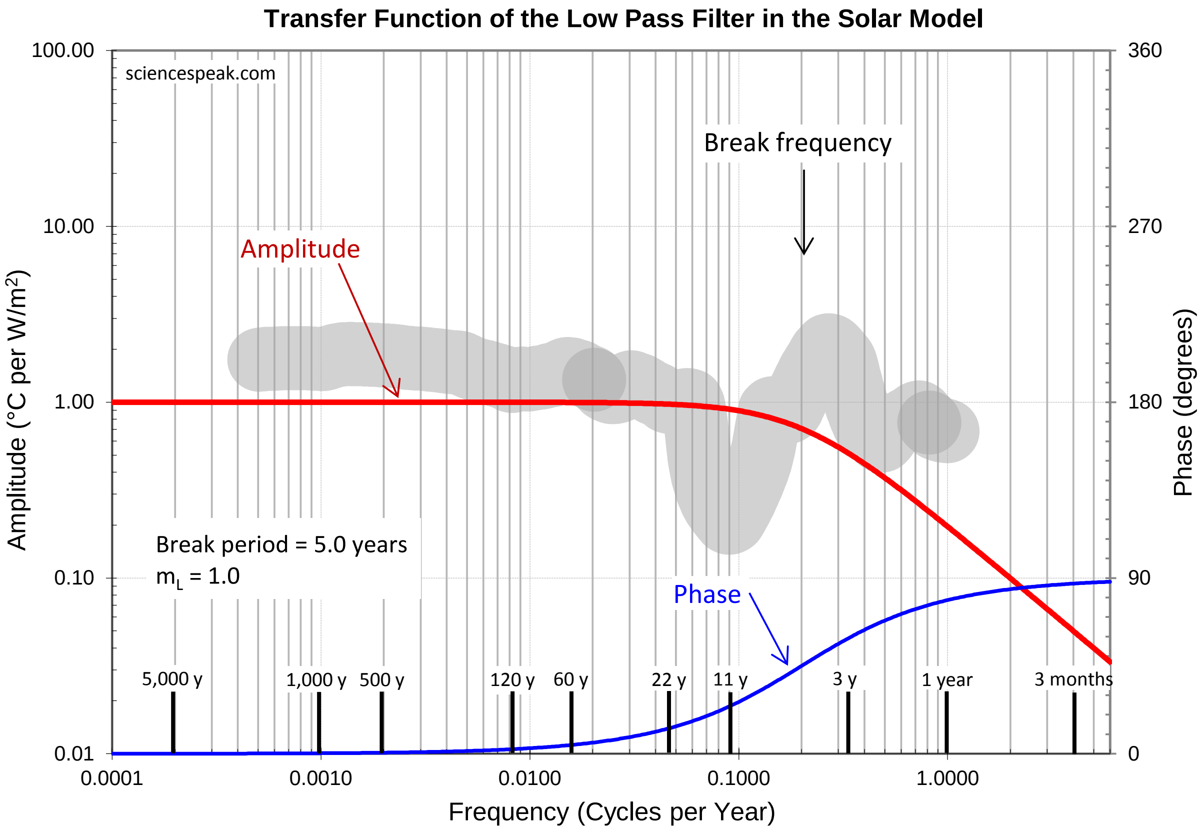

Figure 6: The transfer function of the low pass filter that governs heat accumulation in our climate system. On a log-log graph it has a single 45° bend in the amplitude at the break frequency, sloping downward at higher frequencies. It is shown here with parameter values from P0 (determined later in section 7), and m_L of 1.

While it is exaggerating to say that a low pass filter is "evident" in the empirical transfer function, one can imagine the amplitude of the empirical transfer function being equal to a product of:

- The amplitude of the transfer function of the notch filter in Figure 3.
- The amplitude of the transfer function of the low pass filter in Figure 6.

The transfer function of two systems in cascade is the product of their transfer functions. If the break frequency of the low pass filter is near the notch frequency of the notch filter (as we shall find it is, below), the features of the two filters overlap and it is hard to see the low pass filter because the notch dominates and obscures the gentle bend in the low pass filter's transfer function. Notice how in the empirical transfer function the amplitudes start to fade away at the higher frequencies—the empirical transfer function is compatible with the presence of a low pass filter in the solar subsystem.

The step response of a low pass filter always starts immediately after the corresponding stimulus, without delay or prescience, so it has no causality complications.

6.4 The RATS Multiplier

The output of the low pass filter is the temperature of the radiating surface, around 255 K. The output of the solar model is the temperature at the surface of the Earth, around 288 K, which is in the interior of the opaque ball that is the Earth as seen from space at infrared frequencies. Any climate model must connect the two. At the electromagnetic frequencies in the atmospheric window the radiating surface coincides with the surface of the Earth, but at the other frequencies it is above the surface of the Earth and the two surfaces are separated only by atmosphere.

All atmospheric processes fully resolve themselves in hours to days. On the other hand the solar model is insensitive to anything occurring on timescales shorter than a year, because it uses deseasonalized TSI data as input. So as far as the solar model is concerned, the corresponding changes in radiating and surface temperatures occur simultaneously. Because the climate model is linear by assumption, the connection between the radiating and surface temperatures must therefore be one of proportionality (because the only linear systems whose step responses are simultaneous steps are multipliers). So the changes in surface air temperature are modeled as equal to the changes in radiating temperature times the RATS multiplier m_R, that is, a multiplier connects the radiating temperature and surface air temperature. "RATS" stands for "Radiative Amplification To Surface".

This result does not rely on either the CO_2 or solar assumptions, requiring only that the changes be on timescales of a year or longer.

Note that the transfer function of a multiplier is a horizontal straight line, which has no effect on the empirical transfer function except to raise or lower the entire line on a log graph. Thus the multiplier leaves no characteristic shape on the empirical transfer function by which its existence can be confirmed.

6.5 The Order of the Filters

There being no other influences discernible from the empirical transfer function or elementary physical theory, the notch-delay solar model so far is simply a path from TSI to temperature that just contains a notch filter, a delay filter, a low pass filter, and the RATS multiplier filter. Their individual transfer functions are multiplied together to form the empirical transfer function, so these four filters must be in cascade. However complex multiplication is commutative, so this doesn't indicate their *order*. For that we turn to physical reasoning.

The combination of low pass filter then RATS multiplier models the climate system from the unreflected TSI to the surface air temperature. But the surface air temperature is also the output of the solar model. Therefore the notch and delay filters cannot go *after* the low pass filter in the progression along the energy path from the incoming solar radiation to temperature at the surface—so they must go *before*.

Alternatively, note that the low pass filter models the system from unreflected TSI to radiating temperature, and the atmospheric processes linking the radiating temperature to the surface air temperature are resolved in hours to days, far too quickly to contain the multi-year delay of the notch and delay combination. So the notch and delay must go *before* the unreflected TSI.

The input to the solar model is TSI at 1 AU, which is also the TSI at the top of the atmosphere (after a trivial rescaling by a factor of four). The input to the low pass filter is the unreflected TSI. The notch-delay solar model so far just consists of the four filters in cascade. The notch and delay filters are indivisibly linked together so their order is irrelevant—they simply go together in either order with nothing in-between. The RATS multiplier goes after the low pass filter. Therefore the notch and delay filters are associated with the reflection of TSI back to space, that is, they are modulating the albedo of Earth—as in Figure 7.

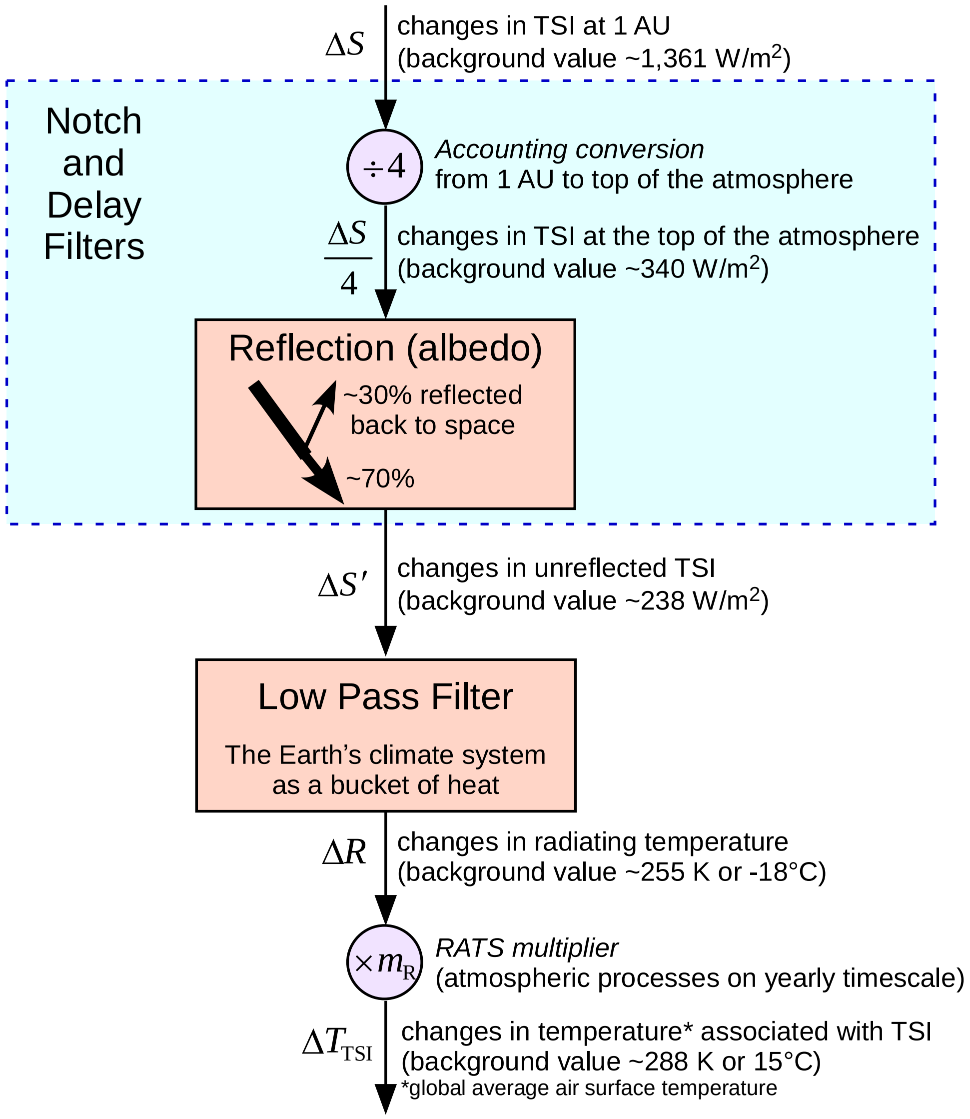

Figure 7: The notch and delay filters modulate the Earth's albedo.

6.6 The Immediate Path

The TSI consist of a background constant S_0 and a changing part ΔS. The development above shows that changes in TSI go through the notch and delay filters to produce changes in the unreflected TSI, after a delay due to the delay filter. But changes in TSI obviously also cause direct and immediate changes in the unreflected TSI, by changing the incoming heat from the Sun. Therefore there are *two* paths by which changes in TSI come to be associated with changes in unreflected TSI:

- The immediate path: Changes in TSI are transmitted through the reflection mechanism of clouds, ice etc., causing immediate changes in unreflected TSI.
- The delayed path: Changes in TSI are associated, after a delay of perhaps several years, with changes in unreflected TSI, and exhibit notching.

The development of the solar model above has evidently been describing only the delayed path. Clearly the model also needs an immediate path, which bypasses the notch and delay filters to produce changes in unreflected TSI, and which therefore shares the same low pass filter and RATS multiplier.

The delayed path was deduced from the empirical transfer function, so presumably the influence on temperature of the delayed path is much greater than the influence of the immediate path or the immediate path would be more evident in that transfer function.

6.7 The Path Multipliers

The transfer functions of the notch, delay, and low pass filters given above were concerned only with the *shape* of their transfer functions, and the scaling has been left underdetermined until now. There are two paths, so let us put a multiplier in each path to scale its input:

- The immediate path multiplier m_{I} scales the changes in TSI on the immediate path.
- The delayed path multiplier m_{D} scales the changes in TSI on the delayed path.

6.8 The Solar Factor

Later, when considering the effects of different relative strengths of CO_2 and solar in causing global warming, we need to be able to scale the output of the solar model. For this purpose we introduce the solar factor s_{F}, a real number from 0 to 1 that multiplies the transfer function of the solar model. We set the solar factor to:

- 0% when causation is nearly all CO_2, so the output of the solar model is zero.
- 100% for nearly all solar causation, in which case the solar model transfer function should be about equal to the empirical transfer function H_{SOCS}.

6.9 The Notch-Delay Solar Model

Putting the elements above together gives the notch-delay solar model, shown in Figure 8.

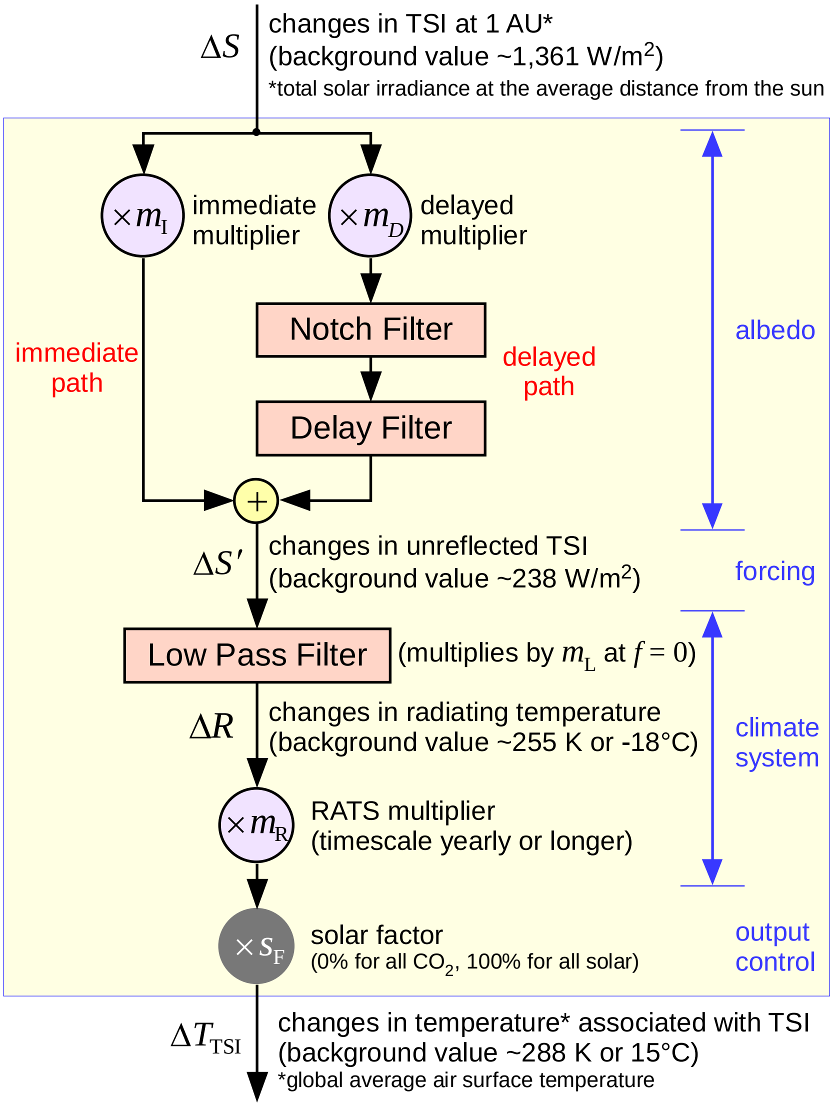

Figure 8: Schematic of the notch-delay solar model.

The notch-delay solar model is defined by its transfer function

$$\begin{aligned} H_{\text{Solar}}(f) &= \left[m_{\text{D}} + m_{\text{N}} H_{\text{Notch}}(f) H_{\text{Delay}}(f) \right] H_{\text{LPF}}(f) m_{\text{R}} s_{\text{F}} \\ &= \left[m_{\text{D}} + m_{\text{N}} \frac{f_{\text{Z}}^2 - f^2 + i2f f_{\text{Z}} \cos\theta_{\text{Z}}}{f_{\text{P}}^2 - f^2 + i2f f_{\text{P}} \cos\theta_{\text{P}}} e^{-i2\pi f d} \right] \frac{m_{\text{R}} m_{\text{L}} s_{\text{F}}}{1 + i(f/f_{\text{B}})}, \quad f \geq 0. \end{aligned} \tag{4}$$

If $\overline{\Delta S}$ is the Fourier transform of the record of TSI changes at 1 AU and $\overline{\Delta T}$ is the Fourier transform of the record of changes in global average surface air temperatures, then

$$\overline{\Delta T}(f) = H_{\text{Solar}}(f) \overline{\Delta S}(f), \quad f \geq 0. \tag{5}$$

The transfer function H_{Solar} is graphed in Figure 9, and its step response in Figure 10.

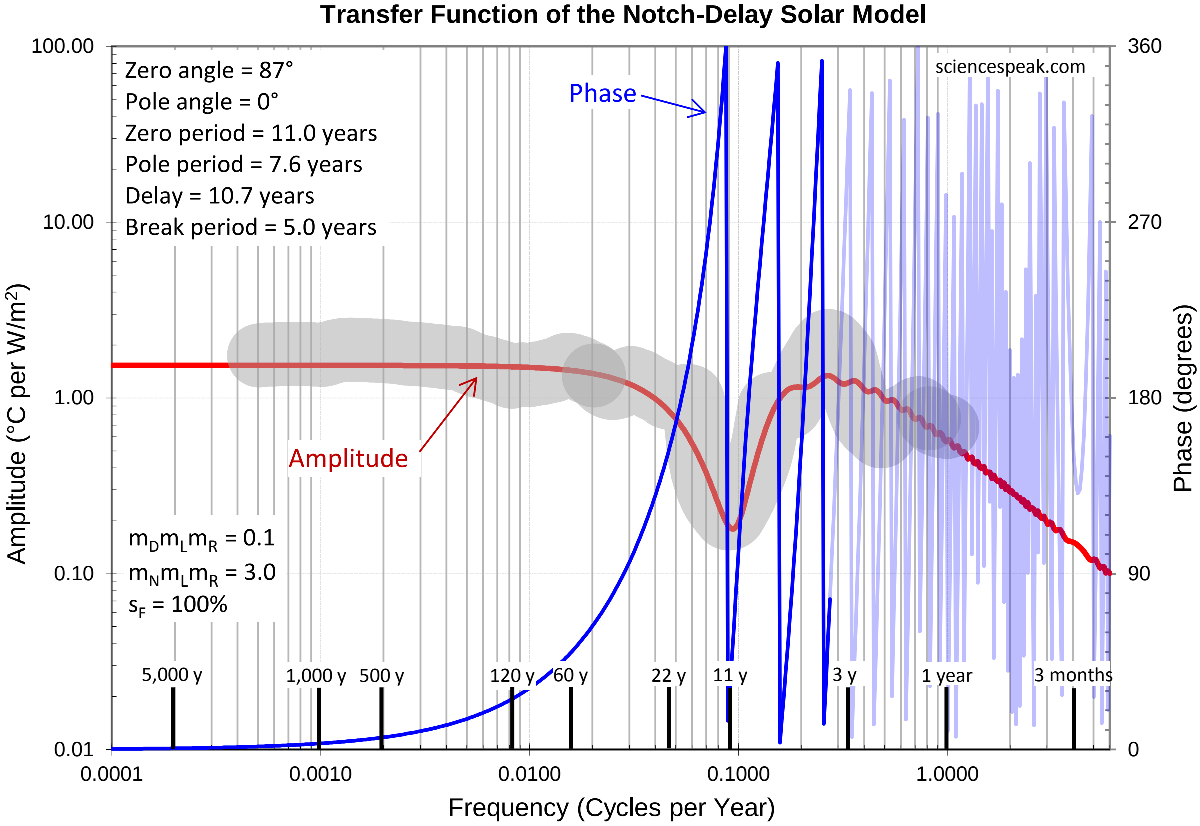

Figure 9: The transfer function of the notch-delay solar model, for parameter values from P0 (determined later in section 7) and a solar factor of 100%. (Technical note: The phase in lighter blue is not graphed correctly because the sampling of the phase for making the graph does not keep up with the ever faster changes as frequency increases.)

The solar model omits a feedback path from the oceans and air to the albedo. While this undoubtedly exists, even for year-to-year changes, it may be a minor effect compared to the pathways in the model. In any case, there is no clear reason for including it.

6.10 Step Response

The time-domain description corresponding directly to equation (4) is a complicated linear differential equation, but fortunately we need never consider it explicitly because we can implement the solar model in the time domain by using the step response (which is obtained from the transfer function. The step response *is* our time-domain description of the solar model.

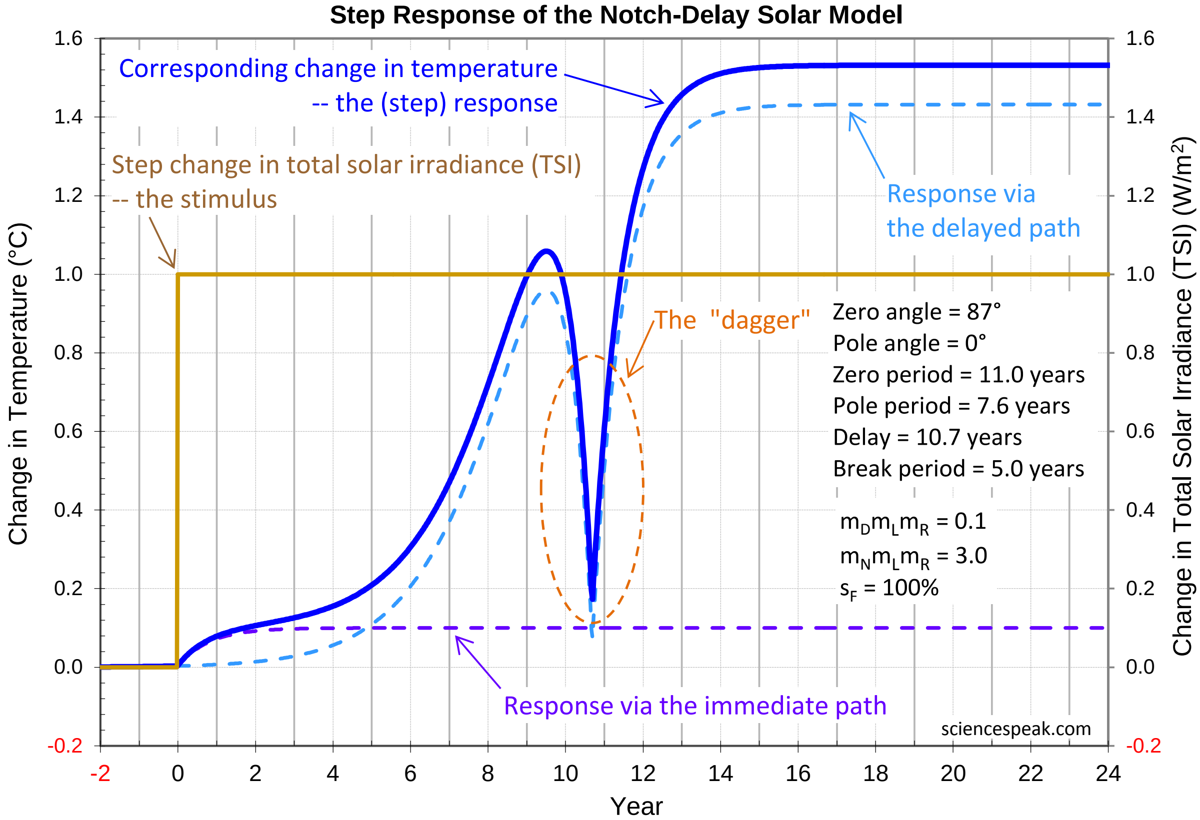

Figure 10: Step response of the solar model (parameter values from P0, as per Figure 9). It is causal. The response is the sum of the response via the immediate path and the response via the delayed path.

The step response of the solar model is shown in Figure 10, as the sum of the step response of the two paths. The P0 parameter values used in Figure 10 are typical of the values that are later found to make the solar model best fit the empirical transfer function and the observed temperatures, so Figure 10 may be taken as representative of the solar model (though the true parameter values might be significantly different).

Due to the delay, the step response of the delayed path is now zero before the stimulus occurs (that is, causal), unlike in Figure 4. The prominent "dagger" in the step response is what causes notching; the delay causes the dagger to arrive after a period roughly equal to the delay (which happens to be close to the notching period in Figure 10).

If there is a sudden step up in TSI at the beginning of year 0, the response of the immediate path is a quick rise in temperature over a couple of years to a small base level that is maintained thereafter. But the step response of the delayed path is larger and more complicated:

- Increases almost imperceptibly for the first three years.
- Increases gradually and at a quickening pace to the middle of year 10.
- Decrease quickly for about a year, about back to the temperature in year 0.
- Increase rapidly then asymptotically over the next five years to level off at the final temperature change, which is reached in year 15 and is about 50% more than the temperature increase achieved before the plunge in year 10.

From the step response we can see that the temperature change for this year, as predicted by the solar model, depends:

- Only a little on the TSI changes in the last five years.
- Somewhat on the TSI changes between 5 and 11 years ago.
- Mainly on the TSI changes that occurred from 11 to 15 years ago.
- Not at all on the TSI changes more than 15 years ago (while this year's temperature depends on all past TSI changes, the temperatures last year and next year are equally affected).

The clean smooth curves and the sharp dagger point in the step response of Figure 10 only exist because that figure shows the step response of the *model*, which is the elegant mathematical construction in equation (4). Naturally a real step response is dirty and messy, with no sharp points. So what is the empirical step response, corresponding to the empirical transfer function? Unfortunately we do not know, because all we have is the amplitude part of the empirical transfer function. The phase part cannot be found by the methods here, and likely cannot be established at all. While the amplitude function of a true TSI or temperature spectrum is a relatively smooth function of frequency for basic physical reasons, the phase functions of their spectra could be highly discontinuous. Thus we cannot smooth or average the phases of the TSI or temperature spectra, and so cannot estimate the true phase part of the empirical transfer function.

We estimated an empirical step response in Figure 11 by using the amplitude part from the empirical transfer function and a smoothed phase part from equation (4). It is messier, with no sharp points.

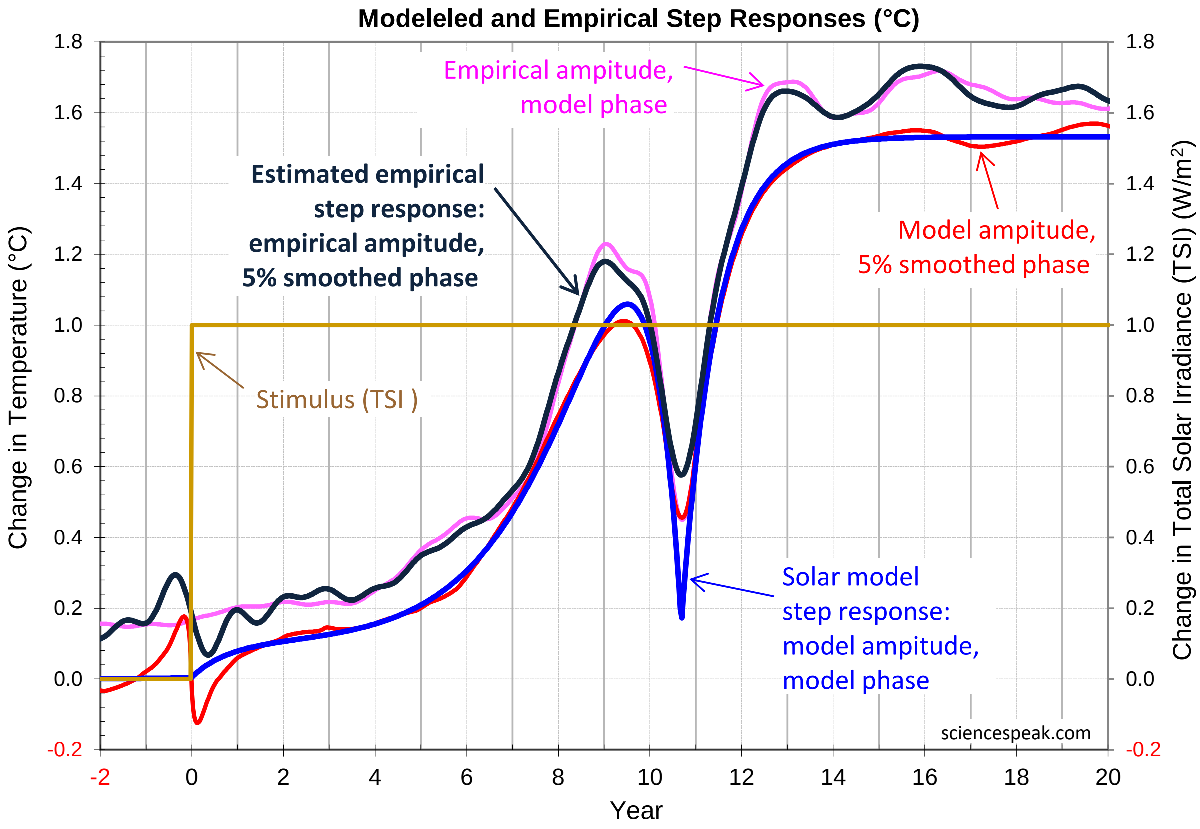

Figure 11: An estimate of the empirical transfer function, corresponding to the amplitude of the empirical transfer function. We do not (and possibly cannot) know the empirical phase function, so we have used the phases from the solar model instead (Figure 9 or equation (4)). We then degraded that phase function by smoothing it, using a window for averaging whose width is 5% of the frequency under consideration. Also shown are the model response (red), and the intermediate responses of partly model and partly empirical (brown and pink). That the responses are slightly non-causal (non-zero before the stimulus) shows the phases are not quite correct.

6.11 Development Notes

In the development of the solar model there is an unavoidable iterative interplay between theory and observation. We couldn't develop the solar model purely from the theoretical side, because the author's theoretical knowledge of climate is insufficient to predict the notch and delay from underlying physical processes. Nor is pure empiricism sufficient, because we cannot see the low pass filter, the RATS multiplier, or the immediate path well enough in the empirical transfer function to justify including them—but the theoretical arguments for including them are strong.

In essence, the solar model contains:

- A notch, because we observe it in the transfer function.
- A delay, because we deduce it is necessary to make the system causal.
- A low pass filter, because we know it must be there and it fits the observations.
- A RATS multiplier, because we deduced this form for the conversion from the radiating temperature to the surface air temperature at the model's timescale.
- An immediate path, because it is obvious theoretically (even though we cannot observe it, because its effect is dominated by the delayed path—see Figure 10).

The form of the solar model is the simplest forms of these elements in the simplest plausible arrangement.

When we started this project, we analyzed the TSI and temperature datasets expecting to find just a low pass filter. The original analysis used the DFT, pairing a TSI dataset with a temperature dataset over exactly the same time period so the frequencies matched, and culling higher frequency sinusoids as noise. It produced a noisy and hazy empirical transfer function, but apparently with no low pass filter—the slope above the break frequency needs to be −1 on the log-log graph, but the line of best fit seemed to be firmly around −0.25. There were no other features apparent, and the project was abandoned.

The project was restarted after we got the idea to weight points in the transfer function diagram by the square root of their amplitude, which further reduces noise and makes the more important points both more influential for the lines of best fits and graphically more obvious. But again, no low pass filter. On the point of abandoning the project again, we noticed that the best data points, the lowest-frequency highest-amplitude points from the PMOD-UAH and HadCrut-Lean pairs, were stubbornly *well below* where the amplitude of the transfer function of a low pass filter should be. Moreover, those points were all at frequencies near 11 years—the average solar cycle length…which could be significant. Then, joining the points with a freehand sketch instead of a straight line, there was a notch *and* a low pass filter—roughly, with a bit of imagination. Could the Earth somehow have a notch filter that filters out the hum of the Sun? Staring at the empirical data, there seemed to be no other conclusion, the data was insisting on it.

In the development of the empirical transfer function, as each processing improvement was made—replacing the DFT by the OFT, weighting points by a power of their amplitude, moving to combined analysis instead of paired analysis, omitting the bonus sinusoids, improving the smoothing curves by using tapered windows—the empirical transfer function thus obtained got progressively closer to the form of the solar model in equation (4). This lends some weight to the correctness of the solar model. If improved signal processing had led instead to progressively greater or randomly fluctuating differences with equation (4), then that equation would likely not be correct. The necessary attention to detail is formidable, which may help explain why this relationship has not been noticed before.

7 Parameter Values of the Solar Model

The details of finding the values of the parameters for the notch-delay solar model are in Appendix L. Only the results, shown in Table 1, are discussed here.

7.1 By Curve Fitting

The probable ranges of the parameters were found by optimization against the empirical transfer function and observed temperatures, guided by a little theory—that is, by curve fitting the solar model to the observed temperatures and TSI, under the solar assumption. The delay was found to be between 10 and 20 years, but the fits were generally better around 11 years so it is more likely around 11 years. Delays below 10 years were ruled out because model projections showed the temperature drop corresponding to the fall in TSI around 2004 would have already start by the end of 2013.

However in our current state of knowledge there is no way of choosing just one representative or "best" set that truly represents the solar model—we don't know the true values, the data is too fuzzy. Nonetheless we are forced to choose one set for some purposes, such as for the graphs of the temperatures computed by the solar model. We arrived at parameter set P25 as the one to represent the solar model to the world, but please be aware that its selection was partly ad hoc and arbitrary, it might not contain the true parameter values, and it wasn't even the set with the best fit to the temperature data.

One additional notation is needed in the model to deal with the various multipliers. Numerical curve fitting can only find the combined multiplier for the whole length of each path from TSI to temperature, not the separate constituent multipliers along the path. So let the DLR multiplier m_{DLR} and NLR multiplier m_{NLR} be defined by

$$m_{\text{DLR}} = m_{\text{D}} m_{\text{L}} m_{\text{R}} \quad \text{and} \quad m_{\text{NLR}} = m_{\text{N}} m_{\text{L}} m_{\text{R}} . \qquad (6)$$

Name	Symbol	Filter	Range	Value in P25	Value in P0	Units
Zero angle	θ_Z	Notch	80 - 87	86.775	87.0	°
Pole angle	θ_P	Notch	0 - 55	0.0	0.0	°
Zero period	$1/f_Z$	Notch	10.3 – 11.6	10.789	11.0	years
Pole period	$1/f_P$	Notch	3.5 – 6.5	7.621	7.6	years
Delay	d	Delay	10 – 20	10.7	10.7	years
Break period	$1/f_B$	Low pass	4 - 25	5.075	5.0	years
DLR multiplier	m_{DLR}		0.05 – 0.30	0.1147	0.10	-
NLR multiplier	m_{NLR}		3 – 14	3.0479	3.00	-
Solar factor	s_F		0 - 100			%

Table 1: Parameter values for the notch-delay solar model. P25 is the set of solar model parameter values used in the climate model outputs graphed below. P0 is a rounded off version of P25, used above to illustrate model development. Although the possible range of the delay is 10 to 20 years, it is most likely around 11 years. See Appendix L.

Our composite temperature data for 1850 to late 1978 comes almost solely from datasets dominated by land thermometer data (HadCrut4, NCDC, and GISTEMP). If those datasets exaggerate the temperature rise over that period, then the sensitivity of the solar model using the parameter sets found here will be exaggerated—because the training data shows too much temperature rise for a given change in TSI. This would mean that the size of the upcoming drop in temperature projected by the solar model is also exaggerated.

The low pass filter was found to have a break period or time constant of about five years, which agrees with what others have found. From [Schwartz, 2012]: "The time constant characterizing the response of the upper ocean compartment of the climate system to perturbations is estimated as about 5 years, in broad agreement with other recent estimates, and much shorter than the time constant for thermal equilibration of the deep ocean, about 500 years."

7.2 The Multipliers

The values of the four individual multipliers can be found, the first two by theoretical considerations alone and the last two from the two combined multipliers found in the curve fitting.

The immediate-path multiplier just converts TSI at 1 AU to TSI at the top of the atmosphere, and applies the background value of albedo. Therefore

$$m_{\mathrm{I}} = \frac{1}{4} \times 70\% = 0.175\,. \tag{7}$$

The LPF multiplier inside the low pass filter can be found by considering long term changes to steady state and applying the Stefan-Boltzmann equation. The value of the low pass filter transfer function in steady state is m_{L} by equation (3) as $f \to 0$, while it is $1/3.1$ by [Stefan-Boltzmann]. Hence

$$m_{\mathrm{L}} = \frac{1}{3.1} = 0.32\,°\mathrm{C} \text{ per } \mathrm{W/m^2}. \tag{8}$$

Now the remaining multipliers can be unraveled:

$$m_{\mathrm{R}} = \frac{m_{\mathrm{ILR}}}{m_{\mathrm{I}} m_{\mathrm{L}}} = \frac{0.1147\,[0.05, 0.30]}{0.175 \times 0.32} = 2.1\,[0.89, 5.4] \tag{9}$$

$$m_{\mathrm{D}} = \frac{m_{\mathrm{DLR}}}{m_{\mathrm{L}} m_{\mathrm{R}}} = \frac{3.0479\,[3.0, 14]}{0.32 \times 2.1[0.89, 5.4]} = 4.5\,[1.7, 49] \tag{10}$$

using the values of m_{ILR} and m_{DLR} in P25 and their ranges (in square brackets above) from Table 1. The ranges of the last two are large because the curve fit is not precise—especially for the immediate-path multiplier, because the influence of the immediate path is dominated by the delayed path.

Appendix K Our Climate Model

This appendix describes the climate model developed for this project, and which generates all the climate modeling in this paper. It does not rely on the solar or CO_2 assumptions.

Our climate model is a "0-D model", dealing only with the global average surface air temperature ("temperature") over time; it does not consider regions or oceans. However it takes only a few seconds to run on a modern pc running Microsoft Windows, and it operates at the same level of resolution as the global TSI and temperature datasets. It is implemented in the Microsoft Excel spreadsheet *Climate.xlsm* using the VBA programming language, and is available for download as described in the *Administration* section near the top of this paper.

The climate model simply sums the temperature changes due to its constituent models: the notch-delay solar model, the CO_2 model, the CFC model, GISS Climate Model-E (the model from the Goddard Institute of Space Studies that published forcings of individual forcing components from 1880 to 2011, such as from TSI, volcanoes, black carbon, snow albedo, or land use, which can be used in our model), and the nukes (atmospheric bomb tests) model. The climate model has a numerical optimizer that, in a few minutes, estimates model parameters that minimize the differences between the climate model output and a specified comparison temperature.

In this paper the climate model is always run from when TSI data becomes available or improves in quality, plus a twenty- year "spin-up" period for the solar model, namely from 1630, 1770, 1900 or 2000. The model runs go slightly into the future to see what is coming up—as discussed in section 6.10, the bulk of the solar influence on today's temperature is from the TSI of 11 to 20 years previously, so the next 10 years of outputs from the solar model are already mostly "locked in". Other inputs are extrapolated in the obvious fashion, such as rising CO_2 levels, though of course the timing and size of future volcanic eruptions are unknown.

K.1 The Four Standard Intervals

There are many climate model runs, optimizing and fitting models to observed data, and comparing the fits for different sets of parameters. Hundreds of years of TSI and temperature data is available, of generally lower quality for further back in time. We need to select a few standard time intervals over which to do our model comparisons.

The TSI data used to drive the solar model is critical. There are four major step-ups in the quality of the TSI data through time:

1. 1610: First yearly data available, in Lean 2000. Proxy data, from sunspots.
2. 1749: Sunspot data available monthly.
3. 1882: Lean 2000 data switches from yearly to monthly.
4. 1978: Satellite observations of TSI and temperature begin.

Prior to 1610 the TSI data is limited to reconstructions from Be10 and C14, and the time resolution is at least five years per data point. The solar model is driven by changes in TSI that undergo a "cycle" every 11 years or so, so this data is inadequate.

As discussed in section 6.10 on the step response of the solar model, the bulk of the solar influence on today's temperature is from the TSI of 11 to 20 years previously, and TSI changes from more than 20 years ago do not affect today's temperature change. Therefore there is a "spin-up" interval of 20 years of TSI data, after which the solar model has all the past TSI data it needs to estimate temperature change. Hence our standard time intervals start about 20 years after the step-ups in the TSI data quality: 1630, 1770, 1900 and 2000. All go to the present.

K.2 Composite TSI and Composite Temperature

To test our climate model against reality we first need to decide "what reality is". We need a single TSI time series to drive the solar model, and a single temperature time series to compare with the output of the climate model.

However our main TSI and temperature datasets disagree with one another and cover different time ranges. All the datasets previously analyzed spectrally were combined into a composite TSI time series and a composite temperature time series, using the following principles:

- During a period where several time series exist, the weighted average of the time series was used. The relative weights were generally those in [previous appendix], except that ACRIM was given the same weight as PMOD after 1992, the Central England temperatures were weighted the same as Moberg, and the land thermometer datasets (HadCrut4, GISTEMP and NCDC) were given a full weighting (1.0, not 0.5).

- The absolute values of the datasets were adjusted so that in periods where datasets overlapped they had the same average changes as the composite dataset.
- When a new dataset entered the composite it was blended in gradually, with a relative weighting rising linearly from zero initially to the full amount after either several data points or several years of monthly data. Likewise when a dataset expired and left the composite it was blended out.

Although the composites were carefully constructed and seem reasonable, they are essentially a compromise between different versions of reality. They are flawed, because they have different spectral properties to the real data: estimating H_{SOCS} by using the composite TSI and composite temperature with the solar-only climate system gives a similar but significantly different H_{SOCS} to the empirical transfer function. But the composites appear to be the best available estimates of TSI and temperature, so they are used to drive and score the models.

K.3 Our Solar Model

The solar model can compute the temperature change due to changes in TSI by either:

- Applying the step response to the composite TSI. Each yearly or monthly change in TSI results in a step response to that change—and the myriad of such step responses are added. No frequency domain techniques are required in this, the basic operation, and nor is there any "look-ahead" at future data, such as by smoothing or transforming to the frequency domain—the model calculates an output temperature point solely from past data values.
  However the spectrum of the composite TSI can be altered by the user, and the composite TSI reconstructed by adding the sinusoidal time series in the altered spectrum. Alterations can:
  - Omit some or all of the bonus sinusoids in the composite TSI. This eliminates most of the pointwise noise in the TSI, which tends to result in slightly closer fits between the solar model and the observed temperature changes.
  - Extend the composite TSI time series forward or back in time, simply by extending the times range of the sinusoidal time series in the reconstructed TSI. This technique should be used sparingly, perhaps only to project TSI forward five or ten years (as discussed in section 6.10 on the step response of the solar model, today's temperature is scarcely influenced by the TSI of the previous 5 years, and only light influenced by the last 10 years of TSI). There is no known reason why projecting forward the TSI by this method should be physically meaningful—there is no general agreement on how the Sun works or on predicting its output.
- Applying the impulse response to the composite TSI. The step response is "more intuitive" in this context. The two approaches give the same results.
- Applying the transfer function of the solar model (equation (4)) to the Fourier transform of the composite TSI (complex multiplication), then adding the sinusoidal time series in the spectrum thus obtained to give the time series of changes in temperature (i.e. inverse Fourier transform). This gives nearly the same outputs as the first two

methods, and would be the same except the sum of the sinusoidal time series in the OFT spectrum is typically not quite equal to the composite TSI.

An alternative approach is to use an adaptive solar cycle lengths. Suppose the step response is not constant, but is stretched or compressed in time so that dagger in Figure 10 lines up exactly with the next solar maximum—for all steps starting around the current solar maximum. However the results were slightly worse than with a fixed length step response.

K.4 Our CO_2 Model

"CO_2" is generally used in this paper to mean the combination of all human-emitted greenhouse gases and aerosols. Good data is only available for carbon dioxide levels, so it is assumed that other gases are emitted roughly proportionally.

The CO_2 model has three parameters for setting the shape and timing of the step response of the CO_2 model based on a two-compartment system (Figure 2 and section 4.4 of [Schwartz, 2012]). A fourth parameter, the equilibrium climate sensitivity (ECS), controls the size of the temperature change—it is proportional to the height of the step response of the CO_2 model at all times. By default, the step response of the fitted CO_2 model reaches 46.5% of the ECS after 20 years (the "transient response") and reaches the full ECS only after 200 years. Good data on carbon dioxide levels from Mauna Loa is available from 1959, and less precise data from Law Dome going back to 1 AD. Thus the changes in temperature due to CO_2 at any time after 1 AD can be estimated for a given ECS (the changes are relatively insensitive to the other three parameters, so long as sensible choices are made).

K.5 Our CFC Model

The recent paper by [Lu, 2013] described a model that estimates temperature changes due to elevated levels of halogenated gases (CFCs), which might be responsible for a significant amount of global warming since 1970. We read the numbers off his graph of estimated temperature changes due to CFCs, which the model scales with an optimizable parameter.

K.6 Our GISS Model

"GISS model" here means the Climate Model E from the Goddard Institute of Space Studies (GISS) at NASA, whose yearly forcings from 1880 through 2011 are published at [Hansen, Sato, Lacis, & Ruedy, 2012]. Those forcings were extrapolated to the present day.

To convert forcings to temperature changes, they were scaled and lagged [Eschenbach, Climate Sensitivity Deconstructed, 2013] [Eschenbach, The Thousand-Year Model, 2013]:

$$\Delta T_n = \lambda(1-a)\Delta F_n + a\Delta T_{n-1} \tag{11}$$

where

- $\Delta T_n = T_n - T_{n-1}$ is the change in temperature from year $n-1$ to year *n*.
- $\Delta F_n = F_n - F_{n-1}$ is the change in total forcing from year $n-1$ to year *n*.
- a is the lag parameter, the delay in feeling the effect of a change in forcing, $a \in [0,1)$. $a = 0$ for no delay and immediate impacts, $a \to 1$ for infinite delay.
- λ is the model's sensitivity (temperature change for a given forcing).

Thus

$$\Delta T_n = \lambda(1-a)\sum_{i=0}^{\infty} a^i \Delta F_{n-i} \tag{12}$$

and the step response (the response to $\Delta F_0 = 1$, $\Delta F_{\text{non}-0} = 0$) is

$$T_n = \lambda(1-a)\sum_{i=0}^{n} a^i \,, \quad n \geq 0 \,. \tag{13}$$

Eschenbach calculated that the GISS model used a lag of $a = 0.711$ [Eschenbach, Climate Sensitivity Deconstructed, 2013]. That lag was used, then λ adjusted as required.

The GISS climate model forcings are partitioned into ten categories:

1. Well-mixed greenhouse gases
2. Ozone
3. Stratospheric water vapor
4. Reflective tropospheric aerosols
5. Aerosol indirect effect
6. Black carbon
7. Snow albedo
8. Volcanoes (stratospheric aerosols)
9. Solar irradiance
10. Land use.

Each category of forcing can be included or not, and individually scaled, in our climate model. For example, the temperature changes due to volcanoes from 1880 can be included in our climate model, at any desired scaling, by including the volcanic forcings from the GISS model. Volcanoes, black carbon, snow albedo, and land use are often brought into our climate model, in the relative strengths they are at in the GISS model, in which case we refer to them collectively as "VBAL".

K.7 Our ENSO Model

ENSO is included as a "model" in our climate model for organizational convenience, even though it is more a leading indicator of temperature changes than a cause of anything. (The ENSO indices correlate very well with the temperature six months later.) The main ENSO indices are included, and can be delayed and scaled to "cause" a temperature change. ENSO is not used in the climate model runs reported here.

K.8 Our Nukes Model

The yield of atmospheric nuclear explosions each year was found in Table 4 (p.207) of [UNSCEAR 2000 REPORT Vol. I, Annex C: Exposures from man-made sources of radiation, 2000]. The logarithms of these yields were converted to temperature changes by scaling and lagging, just as in the GISS climate model in equations (11) to (13).

K.9 All Climate Model Outputs are One-Year Smoothed

All of the outputs of the climate models, and the temperatures they are compared to, are one-year smoothed, because:

- The TSI data is deseasonalized, at 1-AU from the Sun, so it cannot be used to discern features at frequencies greater than one cycle per year. The solar model outputs are thus also limited to frequencies less than one cycle per year.
- Much of the pointwise noise in the climate datasets only lasts a few months.

The smoothing works by replacing each datapoint in a time series with the average of the time series over a one year period centered on the time of the datapoint.

K.10 All Climate Model Outputs have Arbitrary Vertical Offsets

All of the outputs of the climate models in the graphs here are about changes, not absolute values. The graphing software sets their vertical offsets so that (in order of implementation):

1. The sum of the temperature changes from the constituent models equals the temperature changes for the climate model.
2. The comparison temperature changes are zeroed at the start of the interval of interest.
3. The average of the unsmoothed climate model temperature changes equals the average of the unsmoothed comparison temperature changes over a specified period, namely 1850–1950 for intervals starting in 1630 and 1770, over 1900 –1950 for intervals starting in 1900, and over 2000–2012 for intervals starting in 2000.
4. Each constituent model has the same offset at the start of the interval of interest.

The offsets on the graphs have no effect on computations, except that the equalization of the climate model output and the comparison temperatures affect the goodness-of-fit metrics.

K.11 Measuring Goodness of Fit

Our climate model has three methods for measuring the goodness-of-fit between the temperature changes that it calculates and the comparison temperature (which is always the composite temperature for the runs reported in this paper):

1. Absolute deviation. The average of the absolute value of the differences between the climate model output and the comparison temperature. Always positive; smaller indicates a better fit; zero for a perfect fit.
2. Standard deviation. The standard deviation of the differences between the climate model output and the comparison temperature. Always positive; smaller indicates a better fit; zero for a perfect fit.
3. R^2. The square of the sample correlation coefficient between the climate model output and the comparison temperature, except that its sign is changed if the sample correlation coefficient is negative (in which case the climate output and comparison temperatures are negatively linearly correlated). Always from −1 to 1; larger indicates a better fit; one is a perfect fit.

The optimizations reported in this paper always minimize the absolute deviation, because that is the statistic that is closest to an intuitive "best fit". The R^2 statistic is traditional, but higher values don't always correspond to an intuitive idea of a "close fit". The statistics are computed from the points exactly as they appear on the graphs.

K.12 Calculated Attributions to CO_2 or Solar

For a climate model output, the statistics "% of warming due to CO_2" and "% of warming due to solar" are often reported. These are the amounts of temperature change from the CO_2 and solar models over the entire period of interest, as a percentage of the climate model's total temperature change over the period of interest. Because some of the warming may be due to other factors such as the VBAL from the GISS model, and there may be net cooling from other factors like nukes, these two percentages typically add to 80–90%.

These statistics are not the same as the solar factor, which is for scaling the temperature changes computed by the solar model.

K.13 Upcoming 5 Year and 10 Year Drops in Temperature

The solar model has a delay of 10 to 20 years, most likely around 11 years. A pronounced weakness in TSI began sometime from 2002 to 2006, and according to the solar model this will result in a corresponding drop in temperature after the period of the delay has elapsed.

The maximum drop in one-year smoothed output from "today" (the current composite temperature goes to the end of July 2013) to the end of 2018 is computed, and called the "five-year drop". There may be a temperature rise in between now and the minimum in that time. Likewise the "ten year drop" is the drop in the temperature computed by the climate model from the end of July 2013 to the minimum before the end of 2023. The signs of the "drops", as for all temperature changes, are positive for increases—so a five year drop of −0.5°C means it will drop by half a degree by the end of 2018 on a one-year smoothed basis.

Appendix L Finding Parameter Values for the Solar Model

The form of the notch-delay solar model is defined by equation (4) in section 6. It contains eight real parameters, all potentially independent of one another. (This does not count the solar factor, because that's just a knob used to control the amount of solar causation in our climate model. It counts the DLA multiplier m_{DLA} and the NLA multiplier m_{NLA}, but not their constituent multipliers, because the later are numerically inseparable by curve fitting.) How can their values be found?

The amplitude of the transfer function of the solar model could be fitted to $|H_{\text{SOCS}}|$ with the solar factor at 100%, and seven of the eight parameters thereby estimated. But because that approach ignores phase information, there would be no estimate of the all-important delay (which only affects phases). The only other data that contains information about the parameter values is the observed temperatures, so that must be brought in as well.

Both the CO_2 model and the solar model in our climate model (Appendix K) could be turned on, and the climate model asked to optimize the parameters of the CO_2 and solar models so as to minimize the differences between the climate model temperatures and the observed temperatures. However the CO_2 model can produce a smooth curve that matches the temperature rise of the last two centuries moderately well, while the solar model can do the same but with the addition of short-term fluctuations that resemble natural variability but whose timing is as often wrong as right. The optimizer will always favor the CO_2 solution, because a smooth curve is a better fit than a smooth curve with added fluctuations whose timing is wrong half the time. This is not necessarily meaningful in a physical sense, but it does prevent this obvious approach from working.

Instead the solar assumption must be applied to find the parameter values for the solar model, and then afterwards the solar assumption must be suspended and the solar factor used to set the level of solar model output. Besides, the form of the solar model was developed under the solar assumption, so the parameter values should also be found under that assumption.

In our climate model the solar model is turned on, with a 100% solar factor. Volcanoes, black carbon, snow albedo and land use (VBAL) are brought in from the GISS climate model, and the nukes model turned on. The CO_2 and CFC models are turned off so that there are no temperature changes due to changes in greenhouse gas levels. The optimizer is set to simultaneously:

- Minimize the difference between the temperature changes computed by the climate model and the observed temperature changes.
- Minimize the distance between the amplitudes of the transfer function of the solar model and the empirical transfer function, that is, between $|H_{\text{Solar}}|$ and $|H_{\text{SOCS}}|$.

The latter optimization target is given higher priority, to ensure that our climate model as a whole is close to the solar-only climate system, in line with the solar assumption. (In *climate.xlsm*, choose a "Find solar model parameters, from YYYY" scenario and optimize.)

No great numerical precision can be expected. The solar model is being driven with mostly proxy TSI data composed from contradictory datasets, optimizing it to fit temperature data of increasingly questionable accuracy prior to 1979, and anyway the solar model contains a notch filter that is as crude and simple as possible while still producing notching.

There are eleven real parameters to optimize: eight in the solar model, the GISS-VBAL scaling, and the scaling and lagging for the nukes. The optimization surface is complicated, and there are many local minima. For some parameters a broad range of values seems to be at or near optimum, while other parameters require quite specific values for optimality. Some parameters correlate well with others at the local minima, so there are fewer degrees of freedom than suggested simply by the number of variables. It is not clear how to find the parameters. In the end a three stage approach was employed, which finds a range of probable values and also one set of parameter values that is probably at least as good as any other.

L.1 Stage 1: The Monkey

Because the optimization surface is complicated, many optimizations and random starting points were used by employing the "monkey" on the "Comparisons" sheet of *Climate.xlsm*. The monkey randomly chooses parameter values as a starting point for an optimization, then optimizes by varying the values in the set. The monkey does this over and over, only keeping the best sets of optimized parameters and discarding the others. Over a hundred optimizations were run by the monkey for each of the four time intervals, keeping about half. In most optimizations the distance between the model temperatures and the observed temperatures was the average absolute difference, but in a quarter or so the "distance" was the R^2 statistic. Many local minima were thus found, most with similar closeness of fits both to the observed temperature and $|H_{\text{SOCS}}|$.

The optimizations for the different time intervals were of different quality and reliability:

- From 2000: The best data, but the interval is too short, with only one sunspot cycle. The optimization fits are weak (sets kept by the monkey: average R^2 of 26%, maximum 53%). This time interval was mainly ignored for finding parameter values.
- From 1900: Has volcanoes data throughout, contains several sunspot cycles but mainly only waxing of the general solar level. Good optimization fits (sets kept by the monkey: average R^2 of 77%, maximum 85%). The most reliable interval for finding parameter values.
- From 1770: Lacks volcanoes data before 1880, but has many sunspot cycles with non-zero sunspots, and exhibits both waxing and waning of the general solar level. Reasonable optimization fits (sets kept by the monkey: average R^2 of 54%, maximum 59%). This interval was also used to find the parameter values.
- From 1630: Volcanoes data missing before 1880, and the period 1660 to 1720 has suspect TSI data because there were no sunspots. The optimization fits are not as good as the 1900 and 1770 interval results (sets kept by the monkey: average R^2 of 44%, maximum 48%). This interval was not used to find parameter values.

Several relationships emerged between the parameters in the optimal sets of parameter values found by the monkey:

- The break period is linearly correlated to the NLA multiplier ($R^2 \sim 86\%$ for the 1990 interval) and the pole period ($R^2 \sim 80\%$).
- The break period is bi-modally distributed, either from 3 to 7 years or from 10 to 25 years. The low pass filter is obscured in the empirical transfer function, so it is not clear what its break frequency is. If the break frequency is to the left of (lower than) the notch (centered on a frequency of ~11 years) then the notch has a lower right shoulder and the delayed-path multiplier needs to be higher, and vice versa.
- The zero period, which mainly determines the frequency of the notch, is linearly correlated to the pole angle ($R^2 \sim 42\%$ for the 1990 interval) and is narrowly clustered around 9.3 to 11.7 years. The pole angle is anywhere from 0° to 70° (the set limits), and its value does not seem to make much difference.
- The zero angle (which partly determines the notch frequency) is mostly in 83° ± 2°, though there were some at 90° (which is physically less realistic—it leads to a very deep and steep notch for the delayed path, which is a viable solution in the optimizer only because the immediate path smooths the notch of the whole solar model).
- The delay does not correlate well with any of the other parameters. Although it ranges from 5 to 20 years, the more reliable values form a tri-model distribution with strong showings centered around 11 and 18 years and moderate strength around 7 years.

- The DLA multiplier does not correlate well with any of the other parameters. Mostly around 0.2, but ranges from 0.0 to 0.6.

Presumably the performance of the solar model is very sensitive to the zero period and zero angle because their values in these local minima are always about the same, but much less sensitive to the pole angle whose optimized values show a wide range.

The solar model thus has essentially three tuneable parameters: delay, break period, and DLA multiplier. The other five parameters either follow from them (pole angle, pole period, and NLA multiplier), or need to be close to a particular value (zero angle and zero period).

The ranges determined for the parameter values are shown in Table 1. First theory and ad hoc exploration were used to set the ranges that were used by the monkey in randomly choosing parameter values and during each optimization. Second, the ranges in Table 1 came from examining the optimal parameter values in the monkey optimizations after omitting outliers, generally the mean value plus and minus one standard deviation for the 1770 and 1900 monkey runs, plus a further restriction that eliminates delays shorter than 10 years as discussed in the next subsection.

L.2 Stage 2: The Survey

Stage 1 gave a reasonable idea of the likely range of parameters, and some of their interdependencies. The second stage surveyed that “optimal parameter space”, by choosing a set of parameter values in each region of the space and testing the resulting solar model against the observed temperatures (still under the solar assumption).

Starting with some “seed” sets, whose values were systematically chosen to be fairly evenly distributed across the optimal parameter space, typical values were chosen for the three tuneable parameters:

- Low pass filter break period: 5, 15, and 20 years.
- Delay: 5.5, 10.5, or 18 years.
- DLA multiplier: 0.2 or 0.5.

For the variables correlated to these, the linear correlations were used to compute corresponding values (the delayed-path multiplier, the notch pole period, the notch pole angle). For the near constant parameters, a single middling value was chosen: 10.7 years for the notch zero period, and 83° for the zero angle.

This survey has 18 combinations of the tuneable variables, so there were 18 seed sets of parameter values. Each seed set was used as the starting point of an optimization, and the resulting parameter set became a “candidate” parameter set. Sometimes an optimization would result in a candidate whose values were not near its seed, in which case the seed set was perturbed slightly and the optimization run again until the candidate was acceptably near the seed.

Thus 18 candidate sets of parameter values for the solar model were found, which systematically survey the optimal parameter space. The candidate sets were tested by setting the solar model to the parameters of the specified candidate, leaving the solar factor at 100%, and optimizing the remaining three parameters (the GISS scaling parameter, and the nukes scaling and lag parameters). (In *climate.xlsm*, choose a “Nearly all solar, from YYYY (reconstructed TSI)” scenario, choose the parameter set, and optimize.)

Parameter set	P1	P4	P7	P5	P2	P8	P3	P6	P9	Units
Zero angle	85.6	83.4	81.3	84.6	84.0	82.9	83.7	83.5	82.4	°
Pole angle	5.8	44.8	57.7	0.0	4.3	50.2	0.2	21.5	41.5	°
Zero period	10.5	10.7	10.7	11.4	10.0	10.7	10.6	10.8	10.5	years
Pole period	7.2	4.3	4.0	4.7	7.6	4.1	8.3	4.5	3.8	years
Delay	8.95	4.73	4.69	10.36	10.66	10.23	19.26	18.31	17.75	years
Break period	5.6	19.3	26.0	15.2	4.1	21.9	4.2	15.0	19.0	years
ILR multiplier	0.083	0.001	0.027	0.069	0.021	0.001	0.115	0.149	0.158	-
DLA multiplier	3.24	10.17	12.18	9.32	2.61	11.09	2.54	9.17	12.06	-
Dist. To H_{SOCS}	0.162	0.159	0.156	0.165	0.192	0.157	0.147	0.139	0.142	-
1630: Absolute deviation	0.267	0.286	0.288	0.288	0.277	0.282	0.249	0.239	0.267	°C
1630: R^2	0.433	0.451	0.448	0.427	0.445	0.415	0.413	0.406	0.433	-
1630: % solar	86	94	93	92	93	90	78	62	86	-
1770: Absolute deviation	0.200	0.206	0.207	0.205	0.203	0.207	0.202	0.209	0.208	°C
1770: R^2	0.584	0.562	0.570	0.558	0.565	0.544	0.553	0.527	0.531	-
1770: % solar	94	92	95	97	97	97	81	68	69	-
1770: 5 year drop	-0.41	-0.31	-0.35	-0.55	-0.70	-0.50				°C
1770: 10 yr drop	-0.43	-0.24	-0.34	-0.85	-0.75	-0.89	-0.41	-0.25	-0.27	°C
1770: Problems	9 E	9 E	E	9	9 E	E?	R	R E?	Ugh	
1900: Absolute deviation	0.132	0.141	0.138	0.134	0.123	0.134	0.110	0.116	0.116	°C
1900: R^2	0.792	0.733	0.756	0.759	0.791	0.747	0.789	0.742	0.748	-
1900: % solar	87	90	90	97	91	97	77	53	56	-
1900: 5 year drop	-0.39	-0.33	-0.37	-0.55	-0.68	-0.51				°C
1900: 10 yr drop	-0.39	-0.26	-0.37	-0.75	-0.71	-0.80	-0.36	-0.23	-0.23	°C
1900: Problems	9 E	9 E	9 E	9 N	59 E?	9	R?	R	R	-
2000: Absolute deviation	0.075	0.125	0.113	0.073	0.073	0.058	0.064	0.095	0.087	°C
2000: R^2	0.040	0.006	0.018	0.223	0.254	0.287	0.297	0.140	0.133	-
2000: % solar	-999	-420	-186	66	22	73	-42	-108	-169	-
2000: 5 year drop	-0.30	-0.29	-0.32	-0.53	-0.63	-0.50				°C
2000: 10 yr drop	-0.30	-0.16	-0.27	-0.73	-0.64	-0.78	-0.29	-0.17	-0.15	°C
Overall fit metric	1.17	0.99	1.04	1.27	1.38	1.31	1.43	1.16	1.17	-

Table 2: Summary of climate model outputs, with no CO_2 or CFC causation, for the nine candidate parameter sets with less-influential immediate paths. Yellow for delays around 11 years, green 18 years, and purple 7 years. Key to “problems”: E = shows the drop in temperature corresponding to the fall in TSI around 2004 too early; R = shows a moderate temperature rise in 2010 – 2013 that did not occur; 5 (9) = too high around 1950 (1990), N = nukes change on the high side. “Overall fit metric” adds all the R^2s and subtracts all the absolute deviations (bigger is better).

Parameter set	P10	P13	P16	P11	P14	P17	P12	P15	P18	Units
Zero angle	86.8	85.8	81.6	90.0	84.2	90.0	83.2	82.8	82.8	°

Pole angle	32.1	0.0	56.8	26.6	15.1	66.7	0.0	0.0	12.3	°
Zero period	10.0	12.7	10.9	10.7	11.2	13.0	10.6	10.7	10.9	years
Pole period	6.4	4.5	4.0	6.7	4.4	3.6	8.4	4.4	4.3	years
Delay	8.00	4.03	4.69	12.24	10.30	10.78	19.27	18.27	18.34	years
Break period	6.0	16.7	27.1	6.4	16.3	21.1	4.1	14.6	15.7	years
ILR multiplier	0.169	0.482	0.001	0.360	0.055	0.984	0.116	0.162	0.200	-
DLR multiplier	3.46	9.44	12.86	3.55	10.28	8.84	2.51	9.73	10.48	-
Dist. To H_{SOCS}	0.163	0.196	0.157	0.235	0.162	0.299	0.149	0.141	0.147	-
1630: Absolute deviation	0.279	0.288	0.287	0.298	0.287	0.287	0.249	0.239	0.237	°C
1630: R^2	0.468	0.448	0.447	0.438	0.425	0.440	0.414	0.410	0.413	-
1630: % solar	95	96	93	94	91	80	78	64	64	-
1770: Absolute deviation	0.200	0.210	0.207	0.213	0.205	0.203	0.202	0.207	0.207	°C
1770: R^2	0.571	0.511	0.570	0.543	0.556	0.532	0.555	0.538	0.536	-
1770: % solar	89	77	95	97	97	92	81	70	67	-
1770: 5 year drop	-0.23	-0.25	-0.36	-0.75	-0.56	-0.13				°C
1770: 10 yr drop	-0.22	-0.33	-0.36	-0.76	-0.86	-0.36	-0.40	-0.26	-0.22	°C
1770: Problems	9 E	E	E	5 9	9 E?	E Erk	R	Ugh	Ugh	
1900: Absolute deviation	0.130	0.159	0.137	0.164	0.117	0.159	0.109	0.113	0.115	°C
1900: R^2	0.769	0.655	0.758	0.746	0.786	0.674	0.793	0.754	0.749	-
1900: % solar	75	85	90	93	97	85	77	57	50	-
1900: 5 year drop	-0.18	-0.29	-0.37	-0.70	-0.56	-0.19				°C
1900: 10 yr drop	-0.16	-0.30	-0.38	-0.71	-0.86	-0.14	-0.35	-0.24	-0.19	°C
1900: Problems	5 9 E	E	9 E	5 9	9	59ER		R	R	-
2000: Absolute deviation	0.082	0.151	0.111	0.093	0.067	0.161	0.061	0.089	0.094	°C
2000: R^2	0.063	-0.022	0.018	0.028	0.245	0.044	0.306	0.163	0.157	-
2000: % solar	377	1,816	-169	-76	67	-318	-45	-96	-93	-
2000: 5 year drop	-0.11	-0.20	-0.33	-0.61	-0.54	-0.11				°C
2000: 10 yr drop	-0.07	-0.21	-0.29	-0.62	-0.75	-0.05	-0.29	-0.16	-0.13	°C
Overall fit metric	1.18	0.78	1.05	0.99	1.34	0.88	1.45	1.22	1.20	-

Table 3: As for Table 2, but for the candidate parameter sets with more-influential immediate paths.

The results are summarized in Table 2 and Table 3. The absolute deviation and R^2 achieved by each set of parameter values over the four intervals are combined into a single overall fit metric. Also shown is the projected temperature drops for the next 5 and 10 years, corresponding to the drop in TSI that took place somewhere from 2002 to 2006. This is the drop measuring from the last observed temperature in the analysis, in July 2013. Also noted is the percentage of the change in temperature during the complete simulation that is due to the solar model—the remaining change is due to other factors (volcanoes, black carbon, snow albedo, land use, nukes) or unexplained.

Also noted are "problems" with the fits of the parameter set to the observed data. The most interesting is the problems the parameter set has in projecting the timing of the temperature drop corresponding to the fall in TSI around 2004. Some parameter sets project the drop too early ("E" in the tables), because as of this writing (at the end of 2013) the drop hadn't started. This issue is mainly about the delay parameter. Studying the survey results, this earliness problem eliminates all the parameter sets with delays less than 9 years, and was sometimes barely apparent in delays between 10 and 11 years. We conclude from this that the delay is at least 10 years.

In a similar vein, some of the parameter sets with longer delays showed a fairly prominent temperature rise around 2010 to 2013 that didn't occur. This doesn't rule the parameter set out of contention, but it does cast doubt on it. We conclude from this that the delay is more likely around 11 years than 20 years.

L.3 Stage 3: Finding the One Parameter Set

The monkey and survey stages estimated the possible ranges of the parameter values, but we need a set of parameters to represent the solar model in the graphs of the temperatures it computes. This is what most people will look at. For some purposes it is not enough to say that the parameter values are only known to likely lie in these ranges; one set of parameter values is needed that represents the model to the world. Unfortunately there is no way currently of choosing just one representative set that truly represents the solar model—its true values are not known, because the data is too fuzzy. Any chosen set may not contain the true values; at this stage the true values are unknown.

Nonetheless we are forced to choose one set. The parameter set P2 was chosen as a starting point, because it has a delay around 11 years and good overall fits (P14 might have been a good starting point too). It was manually tweaked, using parameter values from other sets that did well, including our favorite parameters from a similar set found in previous research on a solar model with no immediate path. In particular, the zero period was changed to 10.7 years, more in line with the empirical transfer function and the average sunspot cycle length. The delay was adjusted between 10.0 and 12.5 years in increments of as little as 0.1 years, holding the other parameter values constant, repeating the procedures that led to Table 2 and Table 3 for each parameter set, and shown in Table 4.

A delay around 10.7 years was found to work best, with lower values often giving a premature drop, and higher values not fitting as well. Setting the delay to 10.7 years gave the P25 set of parameter values, which is the single set used to show off the solar model. It is emphasized that choosing this particular set was ad hoc and arbitrary, and the true values may well lie elsewhere in the range of possible parameters.

Parameter set	P21	P22	P23	P24	P25	P26	P27	P28	P29	Units
Zero angle	86.8	86.8	86.8	86.8	86.8	86.8	86.8	86.8	86.8	°
Pole angle	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	°
Zero period	10.8	10.8	10.8	10.8	10.8	10.8	10.8	10.8	10.8	years
Pole period	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6	years
Delay	10.01	10.30	10.50	10.60	10.70	10.80	10.90	11.50	12.50	years

Break period	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	years
ILR multiplier	0.115	0.115	0.115	0.115	0.115	0.115	0.115	0.115	0.115	-
DLR multiplier	3.05	3.05	3.05	3.05	3.05	3.05	3.05	3.05	3.05	-
Dist. To H_{SOCS}	0.157	0.161	0.163	0.165	0.166	0.167	0.169	0.176	0.191	-
1630: Absolute deviation	0.285	0.285	0.287	0.288	0.288	0.289	0.289	0.285	0.284	°C
1630: R^2	0.455	0.451	0.454	0.452	0.450	0.449	0.447	0.438	0.440	-
1630: % solar	94	95	96	95	94	94	93	94	91	-
1770: Absolute deviation	0.202	0.203	0.203	0.203	0.203	0.203	0.203	0.204	0.206	°C
1770: R^2	0.576	0.575	0.573	0.572	0.571	0.570	0.568	0.562	0.561	-
1770: % solar	96	96	97	97	97	97	97	97	97	-
1770: 5 year drop	-0.54	-0.60	-0.63	-0.65	-0.66	-0.68	-0.69	-0.73	-0.73	°C
1770: 10 yr drop	-0.67	-0.72	-0.75	-0.76	-0.77	-0.78	-0.79	-0.81	-0.75	°C
1770: Problems	9 E	9 E	9 E	E?	9 E?	9	9 E??	5 9	5 9	
1900: Absolute deviation	0.126	0.125	0.125	0.133	0.133	0.134	0.135	0.130	0.144	°C
1900: R^2	0.779	0.775	0.778	0.789	0.789	0.787	0.785	0.764	0.772	-
1900: % solar	94	95	94	94	94	94	95	97	94	-
1900: 5 year drop	-0.55	-0.60	-0.62	-0.64	-0.64	-0.65	-0.65	-0.65	-0.63	°C
1900: 10 yr drop	-0.63	-0.67	-0.68	-0.69	-0.69	-0.69	-0.69	-0.66	-0.63	°C
1900: Problems	9 E?	59 E?	59 E?	5 9	9	5 9	5 9	5 9 N	5 9	-
2000: Absolute deviation	0.053	0.058	0.064	0.067	0.069	0.071	0.072	0.071	0.085	°C
2000: R^2	0.322	0.363	0.351	0.326	0.301	0.281	0.245	0.097	0.037	-
2000: % solar	-17	9	20	25	29	31	34	27	-42	-
2000: 5 year drop	-0.50	-0.55	-0.57	-0.59	-0.60	-0.60	-0.61	-0.63	-0.58	°C
2000: 10 yr drop	-0.54	-0.58	-0.61	-0.62	-0.63	-0.63	-0.64	-0.63	-0.58	°C
Overall fit metric	1.47	1.49	1.48	1.45	1.42	1.39	1.35	1.17	1.09	-

Table 4: As for Table 2, but for parameter sets considered in the ad hoc search for "the one" (namely P25).

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